Formal Languages

- Sentence Logic and Predicate Logic are formal languages.
- A formal language is a set of sentences generated by rules of formation from a vocabulary.
- The sentences of Sentence Logic and Predicate Logic are not part of natural language (though some may resemble natural-language sentences).
- The formal languages Sentence Logic and Predicate Logic are the objects of our study, and as such they are called object languages.

The Metalanguage

- If we are going to state anything about an object language, we must make use of a language.
- We call a language used to study an object language a metalanguage.
- In theory, the metalanguage may be identical or include the object language.
  - In some parts of linguistics, English is used to describe features of the English language.
- We will strictly separate our metalanguage (English with some extra technical vocabulary) from our object languages.
- Keeping the languages separate allows us to avoid paradoxes of self-reference, such as those described by Tarski.

Use and Mention

- When we employ the metalanguage to refer an item of language, we are said to mention that item of language.
- To mention an item in the object language, one places it within single quotation marks.
– ‘Obama’ has five letters and starts with an ‘O.’
– ‘Barack Obama was born in Ohio’ is a false sentence.
– ‘This sentence is false’ is true.

• In each of the examples, English as the metalanguage is used to mention words or sentences of English.
• In the sentence ‘Barack Obama is president,’ ‘Barack Obama’ is used to refer to the president.

Metavariables
• We may also use English as a metalanguage to mention items of Sentence Logic and Predicate Logic.
  – ‘⊃’ is a connective of Sentence Logic.
  – ‘P ⊃ Q’ is a conditional.
  – If ‘P’ and ‘P ⊃ Q’ have the value t, then ‘Q’ has the value t.
• To state general facts about Sentence Logic and Predicate Logic, we must use expressions that designate classes of items of the object language.
• Such expressions are called metavariables.

Metavariables for Sentences and Sets of Sentences
• To mention metavariables themselves, single quotation marks must be used.
• ‘Q’ through ‘Z’ will be used as metavariables for sentences of both Sentence Logic and Predicate Logic.
• ‘X’ through ‘Z’ will be used as metavariables for sets of sentences of both Sentence Logic and Predicate Logic.

Metavariables and Connectives
• In English, we refer to connectives of Sentence Logic using such expressions as ‘sign of negation’ and ‘sign of the conditional.’
• We may also refer to them by putting single quotation marks around the symbols for the connectives.
  – ‘∼’
  – ‘⊃’
• For conciseness, we must have a way to combine our use of metavariables with reference to connectives, to avoid awkward expressions like the following:
  – If X is a sentence, then the sentence X preceded by ‘∼,’ with the resulting expression enclosed in parentheses, is a sentence.
Names of Themselves

• We would like to be able to combine the use of metavariables with the use of connectives.
  – If X is a sentence, then ¬X is a sentence.
• But such a sentence of the metalanguage includes an item of the object language which is not enclosed in parentheses.
• We will permit such usage (following Carnap) by declaring that the connectives are names, in the metalanguage, of themselves in the object language.
• Names which name themselves are said to be used autonomously.

Metavariables for Predicate Logic

• The vocabulary of Predicate Logic is an extension of the vocabulary of Sentence Logic.
• It contains all the expressions of the vocabulary of Sentence logic, along with other expressions which are proper to Predicate Logic.
• The metavariables for these new vocabulary items will be introduced when the vocabulary items themselves are introduced.

Sets

• An important part of the metalanguage for Sentence Logic and Predicate Logic is set-theoretic notation.
  – We have already described metavariables such as ‘X’ that refer to sets of sentences in the object language.
• Sets are indicated by enclosing the names of their members in curly brackets ‘{’ and ‘}.’
• ‘{‘P,’ ‘P ⊃ Q’}’ indicates the set consisting of the sentences ‘P’ and ‘P ⊃ Q.’
• Ordered sets are indicated by enclosure in angle brackets ‘⟨’ and ‘⟩.’
  – Thus, ‘⟨‘P,’ ‘P ⊃ Q’⟩’ indicates the set consisting of ‘P’ and ‘P ⊃ Q,’ in that order.
Specifying Sets

- There are two methods of specifying the contents of a set.
- The method of *enumeration* simply lists the contents of the set using linguistic items that refer to the objects in that set.
  - \{The President of the United States, Timothy Geithner, 2010\}.
  - \langle 1,2 \rangle
- The method of *description* states what conditions must be satisfied for something to be a member of the set.
  - \{x: x is even\} (the set of all x such that x is even, or the set of all that is even).
  - \{\langle x, y \rangle: x > y\} (the set of all ordered pairs x and y such that x is greater than y.)