Lewis on Possibilia

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Modal Sentences

- Since Aristotle, philosophers have tried to understand the "logic" of modal sentences, particularly sentences of the following kind:
  - It is necessary that *p*,
  - It is possible that *p*,
  - It is contingent that *p*.
- A *modal logic* is a system of rules for determining inferential relations holding among modal sentences (as well as non-modal sentences), for example:
  - If it is necessary that *p*, then it is possible that *p*,
  - If it is necessary that *p*, then *p*.
- Generally, modal logics treat necessity and possibility as "duals":
  - It is necessary that p if and only if it is not possible that not-p,
  - It is possible that p if and only if it is not necessary that not-p.

Boxes and Diamonds

- The standard way of representing modal operators in modal logic is through the use of boxes and diamonds.
- The necessity of *p* is represented by placing a box before '*p*':
  - $\Box p$.
- The possibility of *p* is represented by placing a diamond before '*p*':
  - $\Diamond p$.
- The boxes and diamonds are merely notational abbreviations and have no meaning in their own right.

Possibilia

- Modalities are of interest only if there is a distinction between what is possible and what is actual.
- In ontology, possibles that may or may not be actual are known as "possibilia."
- Two main kinds of possibilia are considered in modal ontology:
  - Possible objects, such as the son of a young couple that is childless by choice,
  - Possible worlds, which consist of possible objects that are capable of existing together.
- Lewis's will make a case that modality should be understood through possibilia rather than through sentential modal operators.

Semantics for Modal Logic

- In the late 1950s, Kripke and others devised interpretations for sentences of modal logic.
- The heart of the semantics is the specification of a set of '*indices*' at which the truth-values of sentences are evaluated.
- For a given set of indices, a *valuation* assigns a truth value for every sentence of a language of modal logic at every index.
- For example, if i is an index, then one valuation might assign *p* the value true at i, while another assigns it false at i.

Relations on Indices

- To evaluate modal sentences for truth-value, we add a two-place (binary) *relation* R on indices.
- A *frame* is the pair consisting of a set of indices and a binary relation on them.
- For example, a frame might consist of the set $\{$i, j, k$\}$ and a binary relation such that iRj, jRk, and iRk.
- In a given frame, the relation R may have certain formal properties.
- In the example, R is transitive: if aRb and bRc, then aRc.
- The only case in which the antecedent holds is where iRj and jRk, and we have it that iRk.

Evaluating Modal Sentences

- Given a frame and a valuation of non-modal sentences, the truth-value of modal sentences can be determined.
- $\Box p$ is true at i if and only if p is true at all indices j such that iRj.
- $\Diamond p$ is true at i if and only if p is true at at least one index j such that iRj.
- The use of 'all' and 'at least one' in these definitions of truth means that the two operators of modal logic can be understood as "quantifiers over the indices of some or another frame, restricted by the relation of that frame" (19)

An Example

- Suppose our frame is as before, with three indices i, j, and k, and R such that iRj, jRk, and iRk.
- Now suppose that on a valuation, p is assigned the value true at i and j, but false at k.
- We get the following results:
  - $\Box p$ is false at i, because p is false at k, to which i is related by R.
  - $\Box p$ is true at j, because p is true at the only index related to i (namely, j).
  - $\Diamond p$ is true at i, because relative to index i is index j, at which p is true.
  - $\Diamond p$ is false at j, because relative to index j is no index at which p is true.

Systems of Modal Logic

- Many systems of modal logic can be generated, according to the restrictions that are placed on the relation R.
- For example, in some systems, $\Box p$ implies $\Box\Box p$.
- Given the semantics for modal logic, this result holds if the relation R is transitive.
- If $\Box p$ is true at index i, then p is true at all indices j related by R to i, which, by transitivity, requires that p be true at all indices k related by R to j.
- So, $\Box p$ is true at all indices j related by R to i, and hence $\Box\Box p$ is true at i, which was to be proved.
- Other sentences of modal logic are validated in the semantics by placing other restrictions on R.
- But which is the "right" system of modal logic?

Modal Logic and Modality

- Strictly speaking, indices and the binary relation on them are merely mathematical devices for evaluating sentences of a formal language.
- The indices and relation might be anything:
  - For example, the indices might be towns and the relation that of being connected by rail.
- If the semantics for modal logic is to indicate anything about modality, the indices in a frame must be "regarded as possible worlds."
- The relation R then relates what is regarded as possible worlds, and we will say that if iRj, then j is "*accessible*" to i.
- Thus, to say that p is necessarily true in a frame is to say that p is true at all accessible worlds in that frame.

Indices and Possible Worlds

- The formal results in modal logic help us to organize our thinking about modal sentences of English, but they decide nothing about which general principles of modality are true.
  - "Metalogical results, by themselves, answer no questions about the logic of modality" (17).
- The results require only indices and a relation on them, not possible worlds and a relation on them.
- Whether $\Box p$ implies $\Box\Box p$ is a matter of whether the relation R on indices is transitive.
- Whether what is necessarily true is necessarily necessarily true depends on whether there is an accessibility relation on possible worlds which is transitive.
- This is a question that can be answered only on the basis of a metaphysical theory of necessity.

The Actual World

- Lewis begins his theory of modality with the actual world, that world in which we live.
- Thus, for Lewis, actuality is an indexical notion: the actual world for an object is the world in which that object exists.
- The world in which we live includes everything in at any distance from where we are or at any time before and after the present.

- All kinds of thing may exist in our world, even objects like spirits which are unknown to physics.
- The only requirement for the actuality of things is they be located at some distance and direction from here and some time before or after now.
- The way things are is just one of many ways in which a world might be.

Modal Realism

- Other worlds exist in ways other than the way in which our world exists.
- The thesis that our world is one among many is called *modal realism*, which is an ontological thesis.
- The other worlds are not related spatially or temporally to our world, but are isolated from it, and from one another.
- There is no causal relation among worlds, nor do they overlap, in the sense that part of one is part of another.
- The only thing that may be in common among worlds is that universals may recur at different worlds.

Possibility

- Each possibility relative to a given world corresponds to what is the case at another world.
- This makes the realm of worlds extremely large.
- The other worlds differ from ours only in what is the case there.
  - They do not differ in terms of what can exist at them.
  - Nor do they differ in "manner of existing."
- To say that only things in our world exist is like using a restricted quantifier.
  - All that exists (relative to us) is in this world.
  - All the beer (that we have) is in the refrigerator.
- Reality encompasses all worlds, and we make none of them.

Motivation

- One reason to believe in a plurality of worlds is that it is serviceable.
- Talk of possibilia has been helpful in clarifying question in many fields of philosophy:
  - Of logic,
  - Of mind,
  - Of language,
  - Of science.
- An analogy is the "paradise" for mathematicians created by accepting set theory, with all the exotic kinds of sets that come with it.
- Set theory achieves this from a theoretically simple starting-point.
- Analogously, granting the existence of a plurality of worlds gives us a fruitful way of understanding possibility from a theoretically simple starting-point, which gives us good reason to think modal realism is true.

Restricted Quantifers

- We can explain possibility relative to our world as existence at another possible world.
  - It is possible that there are blue swans.
  - There is a possible world W in which swans are blue in W.
- The relativizing phrase 'in W' is a restriction on the use of a quantifier like the phrase 'in Australia.'
  - Some swan in W is blue.
  - Some swan in Australia is warm in December.
- "More often than not, modality is *restricted* quantification, and restricted from the standpoint of a given world, perhaps ours, by means of so-called 'accessibility' relations" (7).

Kinds of Modalities

- There are many varieties of modalities, for example:
  - Nomological, what is necessary or possible by virtue of "laws of nature,"
  - Historical, what is necessary or possible by virtue of what has already occurred,
  - Epistemic, what is necessary or possible by virtue of what is known,
  - Deontic, what is necessary or possible by virtue of laws governing action.
- Each type of modality must be understood in terms of its own special relation of accessibility.

Accessibility Relations

- We may describe accessible worlds for the various kinds of modalities:
  - Nomological: all the laws of nature holding at the home world are obeyed,
  - Historical: everything that has happened up the present in the home world is matched by what happens there,
  - Epistemic: everything known at the home world is known is matched by what is known there,
  - Deontic: Everything that is permitted at the home world is permitted there.
- "Restricting of modalities by accessibility . . . relations, like the restriction of quantifiers in general, is a very fluid sort of affair, inconstant, somewhat indeterminate, and subject to instant change in response to contextual pressures" (8).

An Application: Predetermination

- An important metaphysical question is whether a given event is predetermined by past events.
- Some have tried to understand predetermination in terms of what could be known or computed in principle (Laplace).
- Others have tried to base their claims about predetermination on an analysis of causality (the event is implied by the combination of past events and causal laws).
- These attempts are "red herrings" (8).
- "It was predetermined at his creation that Adam would sin iff he does so at every world that both obeys the laws of our world and perfectly matches the history of the world up through the moment of Adam's creation" (8).

Modality *De Re*

- It is more difficult to deal with *de re* modalities, which apply to things rather than sentences.
- We want to say that it is possible that Hubert Humphrey won the 1968 presidential election, which he lost to Richard Nixon.
  - There is an accessible world in which Humphrey satisfies the open formula 'x won the 1968 election.'
- Because possible worlds do not overlap, we cannot say that Humphrey himself won the election at the accessible world.
- A simple solution is to say that Humphrey satisfies 'x won the election' *in absentia*.
- A more complex, but equivalent, solution is to say that there is a counterpart of Humphrey who won the election at the accessible world.

Essential Properties

- We want to say that Humphrey is essentially human.
- If we understand essential properties in modal terms, this is to say that necessarily, Humphrey is human.
- In modal logic, this is equivalent to saying that it is not possible that Humphrey is not human.
- In counterpart theory, this means that there is no accessible world W in which Humphrey has a counterpart in W who satisfies the open formula 'x is not human.'
- This approach can be generalized to all essential properties.

Necessary Existence

- The account of essential properties just given runs into the problem that it seems to require the necessary existence of what has an essential property.
- Consider the open formula 'x does not exist.'
- It seems true that there is no accessible world W in which Humphrey has a counterpart in W who satisfies the open formula 'x does not exist.'
- All counterparts exist at the worlds where they are counterparts.
- This means, in turn, that it is necessary that Humphrey exists, which is "wrong" (11).
- What we want to say is that Humphrey exists necessarily just in case he has a counterpart in every accessible world.

A Problem

- It is all well and good to say that something satisifies 'necessarily Φx' (and so, 'necessarily x exists') if and only if it has a counterpart that satisfies 'Φx' in every accessible world.
- But this raises a problem, given the interdefinability of box and diamond in modal logic.
- Something satisfies 'possibly Φx' if and only if it is not the case that at every world it has some counterpart which satisfies 'x is not Φ.'
- Since Humphrey does not necessarily exist, at some worlds he has no counterparts, and so none that satisfy 'x is not Φ.'
- In that case, Humphrey would possibly be a cat!

A Solution

- The problem of reconciling essential properties and contingent existence arises from the very nature the standard semantics for modal logic.
- A solution would be to abandon the semantics for modal logic and "use the resources of modal realism *directly* to say what it would mean for Humphrey to be essentially human, or to exist contingently" (13).
- For Humphrey to be essentially human is for all of his counterparts in accessible worlds to be human.
- For Humphrey to exist contingently is for there to be accessible worlds at which there is no counterpart of Humphrey.
- What we can abandon in standard semantics of modal logic is understanding 'necessarily' in terms of 'not possibly not.'

Supervenience

- An example of the superiority of modal realism over standard modal semantics is in the analysis of the concept of supervenience.
- "The idea is simple and easy: we have supervenience when there could be no difference of one sort without differences of another sort" (14).
- But the literature is full of alternative non-equivalent definitions.
- Lewis attributes the problem to understanding supervenience in terms of standard modal semantics.
- Supervenience is easily expressed in counterpart theory:
  - Sort A supervenes on sort B if and only if there is no accessible world in which things of sort A are different without there being a difference there among things of sort B.

Superveninence of Physical Laws

- The most common claim about supervenience is that mental properties supervene on physical properties.
- Lewis's discussion of this kind of supervenience is too complex to discuss here, so we will turn to another example.
- It seems possible that laws, chances, and causal relationships supervene on the qualities of points or point-sized bits of matter in space-time.
- Since causal laws, etc. operate on the level of worlds, the thesis would be that no two worlds could differ in their laws without differing in the qualities of their point-constituents.

- Modal realism would understand this as meaning that there are no two (accessible) worlds which differ nomologically without differing in the qualities of their point-constituents.

Supervenience and Standard Modal Semantics

- In standard modal semantics, the thesis that no worlds could differ in their laws without differing in the qualities of their point-constituents would have to be understood as follows:
  - "It is not the case that, possibly, two worlds differ in their laws without differing in their distribution of local qualitative character" (16).
- According to the semantics, this means:
  - There is no world at which two worlds differ in their laws without differing in their distribution of local qualitative character.
- But this last condition is trivial, since at no world do two worlds differ.
- "At any world W, there is only the one single world W" (16).

Modal Realism and Quantifiers

- In ordinary use of language, we restrict the range of our quantifiers to this world, as in the following claim:
  - There are no differences in laws without differences in local qualitative character.
- Supervenience claims are modal:
  - There could be no difference in laws without difference in local qualitative character.
- The effect of modal realism is to interpret 'could' as connecting directly the quantifier 'there is' and the two worlds being compared:
  - There is no difference in laws at any two worlds without difference in local qualitative character at those worlds.

Standard Modal Logic and Quantifiers

- The effect of standard modal semantics is to interpret 'could' as the sentential operator 'it is possible that.'
- Taking indices as possible worlds, to say that there could be no differences in laws without difference in local qualitative character is to be understood as:

  - There is no world at which there is a difference, in any two worlds, in laws without difference in local qualitative character.

- The problem here is that a new quantifier 'there is' must be added before the 'there is' that applies to the world.
- This new quantifier makes unintelligible the unrestricted use of the second quantifier.
- "The moral is that we'd better have other-worldly things to quantify over—not just a primitive modal modifier of sentences" (17).