

External and Internal Relations*

G. E. Moore

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In the index to *Appearance and Reality* (First Edition) Mr. Bradley declares that *all* relations are “intrinsic”; and the following are some of the phrases by means of which he tries to explain what he means by this assertion. “A relation must at both ends *affect*, and pass into, the being of its terms” (p. 364). “Every relation essentially penetrates the being of its terms, and is, in this sense, intrinsic” (p. 392). “To stand in a relation and not to be relative, to support it and yet not to be infected and undermined by it, seems out of the question” (p. 142). And a good many other philosophers seem inclined to take the same view about relations which Mr. Bradley is here trying to express. Other phrases which seem to be sometimes used to express it, or a part of it, are these: “No relations are purely external”; “All relations qualify or modify or make a difference to the terms between which they hold”; “No terms are independent of any of the relations in which they stand to other terms” (See *e.g.*, Joachim, *The Nature of Truth*, pp. 11, 12, 46).

It is, I think, by no means easy to make out exactly what these philosophers mean by these assertions. And the main object of this paper is to try to define clearly one proposition, which, even if it does not give the whole of what they mean, seems to me to be always implied by what they mean, and to be certainly false. I shall try to make clear the exact meaning of this proposition, to point out some of its most important consequences, and to distinguish it clearly from certain other propositions which are, I think, more or less liable to be confused with [41] it. And I shall maintain that, if we give to the assertion that a relation is “internal” the meaning which this proposition would give to it, then, though, in that sense, some relations are “internal,” others, no less certainly, are not, but are “purely external.”

To begin with, we may, I think, clear the ground, by putting on one side two propositions about relations, which, though they seem sometimes to be confused with the view we are discussing, do, I think, quite certainly not give the whole meaning of that view.

The first is a proposition which is quite certainly and obviously true of all relations, without exception, and which, though it raises points of great difficulty, can, I think, be clearly enough stated for its truth to be obvious. It is the proposition that, in the case of any relation whatever, the kind of fact

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which we express by saying that a given term A has that relation to another term B, or to a pair of terms B and C, or to three terms B, C, and D, and so on, in no case simply consists in the terms in question *together with* the relation. Thus the fact which we express by saying that Edward VII was father of George V obviously does not simply consist in Edward, George, *and* the relation of fatherhood. In order that the fact may be, it is obviously not sufficient that there should merely be George and Edward and the relation of fatherhood; it is further necessary that the relation should *relate* Edward to George, and not only so, but also that it should relate them in the particular way which we express by saying that Edward was father of George, and not merely in the way which we should express by saying that George was father of Edward. This proposition is, I think, obviously true of all relations without exception; and the only reason why I have mentioned it is because, in an article in which Mr. Bradley criticizes Mr. Russell (*Mind*, 1910, p. 179), he seems to suggest that it is inconsistent with the proposition that any relations are merely external, and because, so far as I can make out, some other [42] people who maintain that all relations are internal seem sometimes to think that their contention follows from this proposition. The way in which Mr. Bradley puts it is that such facts are unities which are not *completely analysable*; and this is, of course, true, if it means merely that in the case of no such fact is there any set of constituents of which we can truly say: This fact is *identical with* these constituents. But whether from this it follows that all relations are internal must of course depend upon what is meant by the latter statement. If it be merely used to express this proposition itself, or anything which follows from it, then, of course, there can be no doubt that all relations are internal. But I think there is no doubt that those who say this do not mean by their words *merely* this obvious proposition itself; and I am going to point out something which I think they always imply, and which certainly does *not* follow from it.

The second proposition, which, I think, may be put aside at once as certainly not giving the whole of what is meant, is the proposition which is, I think, the natural meaning of the phrases “All relations modify or affect their terms” or “All relations make a difference to their terms.” There is one perfectly natural and intelligible sense in which a given relation may be said to modify a term which stands in that relation, namely, the sense in which we should say that, if, by putting a stick of sealing-wax into a flame, we make the sealing-wax melt, its relationship to the flame has modified the sealing-wax. This is a sense of the word “modify” in which part of what is meant by saying of any term that it is modified, is that it has actually undergone a change: and I think it is clear that a sense in which this is part of its meaning is the only one in which the word “modify” can properly be used. If, however, those who say that all relations modify their terms were using the word in this, its proper sense, part of what would be meant by this assertion would be that all terms which have relations at all actually undergo [43] changes. Such an assertion would be obviously false, for the simple reason that there are terms which have relations and which yet never change at all. And I think it is quite clear that those who assert that all relations are internal, in the sense we are concerned with, mean by this

something which could be consistently asserted to be true of all relations without exception, even if it were admitted that some terms which have relations do not change. When, therefore, they use the phrase that all relations “modify” their terms as equivalent to “all relations are internal,” they must be using “modify” in some metaphorical sense other than its natural one. I think, indeed, that most of them would be inclined to assert that in every case in which a term A comes to have to another term B a relation, which it did not have to B in some immediately preceding interval, its having of that relation to that term causes it to undergo some change, which it would not have undergone if it had not stood in precisely that relation to B; and I think perhaps they would think that this proposition follows from some proposition which is true of all relations, without exception, and which is what they mean by saying that all relations are internal. The question whether the coming into a new relation does thus always cause some modification in the term which comes into it is one which is often discussed, as if it had something to do with the question whether all relations are internal; as when, for instance, it is discussed whether knowledge of a thing alters the thing known. And for my part I should maintain that this proposition is certainly not true. But what I am concerned with now is not the question whether it is true, but simply to point out that, so far as I can see, it can have nothing to do with the question whether all relations are internal, for the simple reason that it cannot possibly follow from any proposition with regard to *all* relations without exception. It asserts with regard to all relational properties of a certain kind, that they have a certain kind of *effect*; and no proposition of this sort [44] can, I think, follow from any universal proposition with regard to *all* relations.

We have, therefore, rejected, as certainly not giving the whole meaning of the dogma that all relations are internal: (1) the obviously true proposition that no relational facts are *completely* analysable, in the precise sense which I gave to that assertion; and (2) the obviously false proposition that all relations modify their terms, in the natural sense of the term “modify,” in which it always has as part of its meaning “cause to undergo a change.” And we have also seen that this false proposition that any relation which a term comes to have always causes it to undergo a change is wholly irrelevant to the question whether *all* relations are internal or not. We have seen finally that if the assertion that all relations modify their terms is to be understood as equivalent to the assertion that all are internal, “modify” must be understood in some metaphorical sense. The question is: What is this metaphorical sense?

And one point is, I think, pretty clear to begin with. It is obvious that, in the case of some relations, a given term A may have the relation in question, not only to one other term, but to several different terms. If, for instance, we consider the relation of fatherhood, it is obvious that a man may be father, not only of one, but of several different children. And those who say that all relations modify their terms always mean, I think, not merely that every different relation which a term has modifies it; but also that, where the relation is one which the term has to several different other terms, then, in the case of *each* of these terms, it is modified by the fact that it has the relation in question to *that* particular

term. If, for instance, A is father of three children, B, C, and D, they mean to assert that he is modified, not merely by being a father, but by being the father of B, also by being the father of C, and also by being the father of D. The mere assertion that all *relations* modify their terms does not, of course, make it quite clear that this is [45] what is meant; but I think there is no doubt that it is always meant; and, I think, we can express it more clearly by using a term, which I have already introduced, and saying the doctrine is that all *relational properties* modify their terms, in a sense which remains to be defined. I think there is no difficulty in understanding what I mean by a *relational property*. If A is father of B, then what you assert of A when you say that he is so is a *relational property*—namely, the property of being father of B; and it is quite clear that this property is not itself a *relation*, in the same fundamental sense in which the relation of fatherhood is so; and also that, if C is a different child from B, then the property of being father of C is a different relational property from that of being father of B, although there is only *one* relation, that of fatherhood, from which both are derived. So far as I can make out, those philosophers who talk of all *relations* being internal, often actually mean by “relations,” “relational properties”: when they talk of all the relations of a given term, they mean all its relational properties, and not merely all the different relations, of each of which it is true that the term has that relation to something. It will, I think, conduce to clearness to use a different word for these two entirely different uses of the term “relation”: to call “fatherhood” a relation, and “fatherhood of B” a “relational property.” And the fundamental proposition, which is meant by the assertion that all relations are internal, is, I think, a proposition with regard to relational properties, and not with regard to relations properly so-called. There is no doubt that those who maintain this dogma mean to maintain that all relational properties are related in a peculiar way to the terms which possess them—that they modify or are internal to them, in some metaphorical sense. And once we have defined what this sense is in which a *relational property* can be said to be internal to a term which possesses it, we can easily derive from it a corresponding sense in which the *relations*, strictly so called, from which relational properties are derived, can be said to be internal.

[46] Our question is then: What is the metaphorical sense of “modify” in which the proposition that all relations are internal is equivalent to the proposition that all relational properties “modify” the terms which possess them? I think it is clear that the term “modify” would never have been used at all to express the relation meant, unless there had been some analogy between this relation and that which we have seen is the proper sense of “modify,” namely, *causes* to change. And I think we can see where the analogy comes in by considering the statement, with regard to any particular term A and any relational property ϕ , which belongs to it, that A *would have been different from what it is if it had not had ϕ* : the statement, for instance, that Edward VII would have been different if he had not been father of George V. This is a thing which we can obviously truly say of A and ϕ , in some sense, whenever it is true of ϕ that it *modified* A in the proper sense of the word: if the being held in the flame causes the sealing-wax to melt, we can truly say (in some sense) that the sealing-wax

would not have been in a melted state if it had not been in the flame. But it seems as if it were a thing which might also be true of A and ϕ , where it is *not* true that the possession of, ϕ *caused* A to change; since the mere assertion that A would have been different, if it had not had ϕ , does not necessarily imply that the possession of ϕ *caused* A to have any property which it would not have had otherwise. And those who say that all relations are internal do sometimes tend to speak as if what they meant could be put in the form: In the case of every relational property which a thing has, it is always true that the thing which has it would have been different if it had not had that property; they sometimes say even: If ϕ be a relational property and A a term which has it, then it is always true that A *would not have been* A if it had not had ϕ . This is, I think, obviously a clumsy way of expressing anything which could possibly be true, since, taken strictly, it implies the [47] self-contradictory proposition that if A had not had ϕ , it would not have been true of A that it did not have ϕ . But it is nevertheless a more or less natural way of expressing a proposition which might quite well be true, namely, that, supposing A has ϕ , then anything which had not had ϕ would necessarily have been different from A. This is the proposition which I wish to suggest as giving the metaphorical meaning of ϕ *modifies* A, of which we are in search. It is a proposition to which I think a perfectly precise meaning can be given, and one which does not at all imply that the possession of ϕ *caused* any change in A, but which might conceivably be true of all terms, and all the relational properties they have, without exception. And it seems to me that it is not unnatural that the proposition that this is true of ϕ and A, should have been expressed in the form, “ ϕ modifies A,” since it can be more or less naturally expressed in the perverted form, “If A had not had ϕ it would have been different,”—a form of words, which, as we saw, can also be used whenever ϕ does, in the proper sense, modify A.

I want to suggest, then, that one thing which is always implied by the dogma that, “All relations are internal,” is that, in the case of every relational property, it can always be truly asserted of any term x which has that property, that any term which had not had it would necessarily have been different from x .

This is the proposition to which I want to direct attention. And there are two phrases in it, which require some further explanation.

The first is the phrase “would necessarily have been.” And the meaning of this can be explained, in a preliminary way, as follows:—To say of a pair of properties ϕ and ψ that any term which had had ϕ would necessarily have had ψ is equivalent to saying that, in every case, from the proposition with regard to any given term that it has ϕ , it *follows* that that term has ψ : *follows* being understood in the sense in which from the [48] proposition with regard to any term, that it is a right angle, it follows that it is an angle, and in which from the proposition with regard to any term that it is red it follows that it is coloured. There is obviously some very important sense in which from the proposition that a thing is a right angle, it does follow that it is an angle, and from the proposition that a thing is red it does follow that it is coloured. And what I am maintaining is that the metaphorical sense of “modify,” in which it is maintained that all relational properties modify the subjects which possess

them, can be defined by reference to this sense of “follows.” The definition is: To say of a given relational property ϕ that it modifies or is internal to a given term A which possesses it, is to say that from the proposition that a thing has not got ϕ it follows that that thing is different from A. In other words, it is to say that the property of not possessing ϕ , and the property of being different from A are related to one another in the peculiar way in which the property of being a right-angled triangle is related to that of being a triangle, or that of being red to that of being coloured.

To complete the definition it is necessary, however, to define the sense in which “different from A” is to be understood. There are two different senses which the statement that A is different from B may bear. It may be meant merely that A is *numerically* different from B, *other* than B, not identical with B. Or it may be meant that not only is this the case, but also that A is related to B in a way, which can be roughly expressed by saying that A is *qualitatively* different from B. And of these two meanings, those who say: That all relations make a *difference* to their terms, always, I think, mean difference in the latter sense and not merely in the former. That is to say, they mean, that if ϕ be a relational property which belongs to A, then the absence of ϕ entails not only numerical difference from A, but qualitative difference. But, in fact, from the proposition that a thing is qualitatively different from A, it does follow that it is also numerically [49] different. And hence they are maintaining that every relational property is “internal to” its terms in both of two different senses at the same time. They are maintaining that, if ϕ be a relational property which belongs to A, then ϕ is internal to A both in the sense (1) that the absence of ϕ entails qualitative difference from A; and (2) that the absence of ϕ entails numerical difference from A. It seems to me that neither of these propositions is true; and I will say something about each in turn.

As for the first, I said before that I think some relational properties really are “internal to” their terms, though by no means all are. But, if we understand “internal to” in this first sense, I am not really sure that any are. In order to get an example of one which was, we should have, I think, to say that any two different qualities are always *qualitatively* different from one another: that, for instance, it is not only the case that anything which is pure red is qualitatively different from anything which is pure blue, but that the quality “pure red” itself is qualitatively different from the quality “pure blue.” I am not quite sure that we can say this, but I think we can; and if so, it is easy to get an example of a relational property which is internal in our first sense. The quality “orange” is intermediate in shade between the qualities yellow and red. This is a relational property, and it is quite clear that, on our assumption, it is an internal one. Since it is quite clear that any quality which were *not* intermediate between yellow and red, would necessarily be *other than* orange and if any quality *other* than orange must be *qualitatively* different from orange, then it follows that “intermediate between yellow and red” is internal to “orange.” That is to say, the absence of the relational property “intermediate between yellow and red,” entails the property “different in quality from orange.”

There is then, I think, a difficulty in being sure that any relational properties

are internal in this first sense. But, if what we want to do is to show that some are not, and that [50] therefore the dogma that all relations are internal is false, I think the most conclusive reason for saying this is that if all were internal in this first sense, all would necessarily be internal in the second, and that this is plainly false. I think, in fact, the most important consequence of the dogma that all relations are internal, is that it follows from it that all relational properties are internal in this second sense. I propose, therefore, at once to consider this proposition, with a view to bringing out quite clearly what it means and involves, and what are the main reasons for saying that it is false.

The proposition in question is that, if ϕ be a relational property and A a term to which it does in fact belong, then, no matter what ϕ and A may be, it may always be truly asserted of them, that any term which had *not* possessed ϕ would necessarily have been other than—numerically different from—A: or, in other words, that A would necessarily, in all conceivable circumstances, have possessed ϕ . And with this sense of “internal,” as distinguished from that which says *qualitatively different*, it is quite easy to point out some relational properties which certainly are internal in this sense. Let us take as an example the relational property which we assert to belong to a visual sense-datum, when we say of it that it has another visual sense-datum as a spatial part: the assertion, for instance, with regard to a coloured patch half of which is red and half yellow: “This whole patch contains this patch” (where “this patch” is a proper name for the red half). It is here, I think, quite plain that, in a perfectly clear and intelligible sense, we can say that any whole, which had not contained that red patch, could not have been identical with the whole in question: that from the proposition with regard to any term whatever that it does not contain *that* particular patch it *follows* that that term is *other* than the whole in question—though *not* necessarily that it is qualitatively different from it. *That* particular whole could not have existed without having that particular patch for a part. But it seems no less clear, at first sight, that there are many [51] other relational properties of which this is not true. In order to get an example, we have only to consider the relation which the red patch has to the whole patch, instead of considering as before that which the whole has to it. It seems quite clear that, though the whole could not have existed without having the red patch for a part, the red patch might perfectly well have existed without being part of that particular whole. In other words, though every relational property of the form “having *this* for a spatial part” is “internal” in our sense, it seems equally clear that every property of the form “is a spatial part of this whole” is *not* internal, but purely external. Yet this last, according to me, is one of the things which the dogma of internal relations denies. It implies that it is just as necessary that anything, which is in fact a part of a particular whole, should be a part of that whole, as that any whole, which has a particular thing for a part, should have that thing for a part. It implies, in fact, quite generally, that any term which does in fact have a particular relational property, could not have existed without having that property. And in saying this it obviously flies in the face of common sense. It seems quite obvious that in the case of many relational properties which things have, the fact that they have them is *a mere matter*

of fact: that the things in question *might* have existed without having them. That this, which seems obvious, is true, seems to me to be the most important thing that can be meant by saying that some relations are purely external. And the difficulty is to see how any philosopher could have supposed that it was not true: that, for instance, the relation of part to whole is no more external than that of whole to part. I will give at once one main reason which seems to me to have led to the view, that all relational properties are internal in this sense.

What I am maintaining is the common-sense view, which seems obviously true, that it may be true that A has in fact got ϕ , and yet also true that A might have existed without having ϕ . And I say that this is equivalent to saying that it [52] may be true that A has ϕ , and yet *not* true that from the proposition that a thing has *not* got ϕ it *follows* that that thing is *other* than A—numerically different from it. And one reason why this is disputed is, I think, simply because it is in fact true that if A has ϕ , and x has not, it *does* follow that x is other than A. These two propositions, the one which I admit to be true (1) that if A has ϕ , and x has not, it *does* follow that x is other than A, and the one which I maintain to be false (2) that if A has ϕ , then from the proposition with regard to any term x that it has not got ϕ , it *follows* that x is other than A, are, I think, easily confused with one another. And it is in fact the case that if they are not different, or if (2) follows from (1), then no relational properties are external. For (1) is certainly true, and (2) is certainly equivalent to asserting that none are. It is therefore absolutely essential, if we are to maintain external relations, to maintain that (2) does *not* follow from (1). These two propositions (1) and (2), with regard to which I maintain that (1) is true, and (2) is false, can be put in another way, as follows: (1) asserts that if A has ϕ , then any term which has not, *must* be other than A. (2) asserts that if A has ϕ , then any term which had not, *would necessarily be* other than A. And when they are put in this form, it is, I think, easy to see why they should be confused: you have only to confuse “must” or “is necessarily” with “would necessarily be.” And their connexion with the question of external relations can be brought out as follows: To maintain external relations you have to maintain such things as that, though Edward VII was in fact father of George V, he *might* have existed without being father of George V. But to maintain this you have to maintain that it is *not* true that a person who was *not* father of George would necessarily have been other than Edward. Yet it is, in fact, the case, that any person who was not father of George *must* have been other than Edward. Unless, therefore, you can maintain that from this true proposition it does *not* follow that any person who was *not* father of George *would* [53] *necessarily* have been other than Edward, you will have to give up the view that Edward might have existed without being father of George.

By far the most important point in connexion with the dogma of internal relations seems to me to be simply to see clearly the difference between these two propositions (1) and (2), and that (2) does *not* follow from (1). If this is not understood, nothing in connexion with the dogma can, I think, be understood. And perhaps the difference may seem so clear, that no more need be said about it. But I cannot help thinking it is not clear to everybody, and that it

does involve the rejection of certain views, which are sometimes held as to the meaning of “follows.” So I will try to put the point again in a perfectly strict form.

Let ϕ be a relational property, and A a term to which it does in fact belong. I propose to define what is meant by saying that ϕ is internal to A (in the sense we are now concerned with) as meaning that from the proposition that a thing has not got ϕ , it “follows” that it is *other* than A.

That is to say, this proposition asserts that between the two properties “not having ϕ ” and “other than A,” there holds that relation which holds between the property “being a right angle” and the property “being an angle,” or between the property “red” and the property “coloured,” and which we express by saying that, in the case of any term whatever, from the proposition that that term is a right angle, it follows, or is deducible, that it is an angle. Let us express the relation which we assert to hold between a particular proposition p , and a particular proposition q , when we say that in this sense q follows from or “is deducible from” p , by the symbol “ent”; which I have chosen to express it, because it may be used as an abbreviation for “entails,” and because “ p entails q ” is a natural expression for “ q follows from p ,” i.e., “entails” can naturally be used as the converse of “follows from.” (We cannot unambiguously use the phrase “ p implies q ” as equivalent to “ q follows [54] from p ,” though it is in fact often so used, because, especially in consequence of Mr. Russell’s writings, “implies” has come to be used as a name for a totally different relation we might perhaps use “ p logically implies q ” or “ p formally implies q ,” though Mr. Russell has also given a different meaning to “formal” implication). “ p ent q ” will then assert that there holds between p and q that relation which holds, for instance, between the two premisses of a syllogism in *Barbara*, taken as one conjunctive proposition, and the conclusion, equally whether the premisses be true or false; and which does not hold, for instance, between the proposition “Socrates was a man” and the proposition “Socrates was a mortal,” even though it be in fact true that all men are mortal. And we can express the assertion that ϕ is “internal to” A, using (I hope correctly) the symbols of *Principia Mathematica*, in addition to our new symbol “ent” by saying that what it asserts is:

$$(x) : \sim \phi x . \text{ent} . x \neq A;^1$$

or, in other words: “for all values of x , the proposition that you get by asserting of a particular value of x , say B, that B has *not* got ϕ , *entails* the proposition that B is other than A.” The assertion with regard to a particular term A and a particular relational property ϕ , which A actually has, that ϕ is internal to A means then: $(x) : \sim \phi x . \text{ent} . x \neq A$. And this is, of course, logically equivalent to: $(x) : x = A . \text{ent} . \phi x$; which is, in its turn, equivalent to “anything which were identical with A, would, in any conceivable universe, necessarily have ϕ ” or to “A could not have existed in any possible world without having ϕ ”; just

¹Modern logic uses parentheses or brackets in place of Russell’s dot notation. A greater number of dots indicates greater scope. With parentheses as punctuation, and with Moore’s ‘ent’ operator added to modern notation, the formula would read: $(\forall x)((\sim \phi x) \text{ent} (x \neq A))$.

as the proposition “In any possible world a right angle must be an angle,” is, I take it, just equivalent to $(x) : x$ is a right angle . ent . x is an angle.

Having thus got what is meant by asserting of a particular term A and a particular relational property ϕ , which A in fact possesses, that ϕ is “internal to” A, we can then express what I am calling the dogma of internal relations, or the dogma that *all* relational properties are internal to the terms which [55] have them, by saying that what it asserts is: that, for all those values of ϕ which are relational properties, the proposition “ $(x, y) : . \phi x :) : \sim \phi y . \text{ent} . y \neq x$ ” is true: or (to give the equivalent form) the proposition

$$“(x, y) : . \phi x :) : y = x . \text{ent} . \phi y.”^2$$

This assertion that, for all those values of ϕ which are relational properties, this proposition is true is what I called proposition (2) above, and is what I maintain to be obviously false. What I maintain to be true is that for *some* values of ϕ , which are relational properties, the proposition

$$“(x, y) : . \phi x :) : \sim \phi y . \text{ent} . y \neq x”$$

is true, and that for others it is false: and, those values of ϕ for which it is true I propose to call “internal relational properties,” those for which it is false “external relational properties.”³

And now let us contrast (2) in this form, with what I called above proposition (1), and which I admit to be true, and which I suggested has led to the assertion of (2) through confusion. What (1) asserts is that, for all values of ϕ , the proposition “ $(x, y) : . \phi x . \sim \phi y : \text{ent} : y \neq x$ ” is true; or (what is logically equivalent to this), the proposition

$$“(x, y) : . \phi x : \text{ent} : \sim \phi y . y \neq x”$$

is true.⁴ In other words, it asserts that, if you take a particular relational property ϕ , and a particular term A which has it, then, whatever ϕ and A may be, the proposition that A has ϕ allows the deduction that, *as a matter of fact*, no term, which is without ϕ , is identical with A. It does not for a moment assert that from the proposition that A has ϕ it follows that no term which did not *could* be identical with A; nor even (which is all that (2) asserts) that in no case is the proposition that a particular term has a particular relational property true, and the proposition, that no term *could* be without that property and yet be identical with the term in question, false. (2) therefore, is neither identical with nor follows from (1). To [56] say that it does follow from it is to say that from $p . q : \text{ent} : r$ it follows that $p :) : q . \text{ent} . r$; which can be easily seen

²Apparently due to typographical limitations, the original article uses the symbol ‘)’ in place of Russell’s ‘ \supset .’ Also, in modern notation, two separate universal quantifiers would be used, as with ‘ $(\forall x)(\forall y)$.’ The centered formula would read: $(\forall x)(\forall y)((Fx \supset (y = x \text{ ent } Fy))$.

³The printed article is inconsistent in the display of three dots. To avoid confusion, the first of the modes of display has been adopted here, as it conforms to Russell’s usage. The other version is ‘ \therefore ’

⁴I.e., $(\forall x)(\forall y)((Fx \text{ ent}(\sim Fy \supset y \neq x))$.

to be false by taking for p and q the two premisses of a syllogism in *Barbara*, and for r the conclusion. The conjunction, “All men are mortal *and* Socrates is a man” does entail “Socrates is mortal.” But it is obviously not the case that there follows from this what “ $p :) : q . \text{ent} . r$ ” asserts; namely, that it is *not* the case that “All men are mortal” is true and the proposition “‘Socrates is a mortal’ follows from ‘Socrates is a man,’” false. The proposition that “Socrates is a mortal” follows from “Socrates is a man” *is* false; and yet “All men are mortal” may quite well be true. Or, to take the alternative form of (1). To say that (2) follows from (1) is to say that from $p : \text{ent} : q .) . r$ there follows the proposition: $p :) : q . \text{ent} . r$. But this again can be easily seen to be false in the same way. The proposition “All men are mortal” does entail that “Socrates is a man *materially implies* (to use Mr. Russell’s expression for) “Socrates is mortal”; that is to say, it entails that it is not the case both that “Socrates is a man” is true, and “Socrates is mortal” false. But it does not in the least follow from this that “All men are mortal” *materially implies* that “Socrates is a mortal” follows from “Socrates is a man”; on the contrary, it may, as we have seen, quite well be the case that “All men are mortal” is true, and yet the proposition that “Socrates is a man” entails “Socrates is a mortal” false.

To maintain, therefore, that (2) follows from (1) is mere confusion. And the source of the confusion is, I think, pretty plain: (1) allows you to assert that, if ϕA is true, then the proposition “ $\sim \phi y .) . y \neq A$ ” *must* be true. And what the “must” here expresses is merely that this proposition follows from the hypothesis ϕA , not that it is *in itself* a necessary proposition. But it is supposed, through confusion, that what is asserted is that, on the hypothesis ϕA , “ $\sim \phi y .) . y \neq A$ ” is *in itself*, a necessary proposition, that is to say, that ϕA materially implies “ $\phi y . \text{ent} . y \neq A$ ”—a thing which is true, if ϕ is an [57] internal relational property, and false if it is an external one. I have here used the phrase “a proposition which is necessary *in itself*,” and have implied a definition of it. The definition may be roughly indicated by saying: “ $\phi x .) . \psi x$ is a proposition that is necessary in itself (or apodeictic),” means “ $\phi x . \text{ent} . \psi x$.” That is to say, I am maintaining that $\phi x) \psi x$ is a necessary truth, if and only if it is also true that $\phi x . \text{ent} . \psi x$. This seems to me to give what has in fact been generally meant in philosophy by “necessary truths,” e.g., by Leibniz; and to point out the distinction between them and mere matters of fact. Using this language, what the dogma of internal relations asserts may, I think, be expressed by saying that it asserts that, on the hypothesis that ϕA is true, ϕA is itself a necessary truth; since ϕA is equivalent to $x = A .) . \phi x$, and it asserts that, on the hypothesis ϕA , $x = A . \text{ent} . \phi x$. I, on the contrary, in asserting that some relational properties are external, am asserting that ϕA is often a mere matter of fact even where it is true; that is to say, that though, where it is true, $x = A) \phi x$, yet this is often *not* a necessary truth, since it is *not* true that $x = A . \text{ent} . \phi x$.

So much for the distinction between (1) which is true, and (2), or the dogma of internal relations, which I hold to be false. But I said above, in passing, that my contention that (2) does not follow from (1), involves the rejection of certain views that have sometimes been held as to the meaning of “follows”; and I think

it is worth while to say something about this.

It is obvious that the possibility of maintaining that (2) does not follow from (1), depends upon its being true that from $(x) : \phi x . \psi x$ the proposition $(x) : \phi x . \text{ent} . \psi x$ does not follow. And this has sometimes been disputed, and is, I think, often not clearly seen.

To begin with, Mr. Russell, in the *Principles of Mathematics* (p. 34), treats the phrase “ q can be deduced from p ” as if it meant exactly the same thing as “ $p \supset q$,” or “ p materially implies q ”; and has repeated the same error elsewhere, *e.g.*, in [58] *Philosophical Essays* (p. 166), where, he is discussing what *he* calls the axiom of internal relations. And I am afraid a good many people have been led to suppose that, since Mr. Russell has said this, it must be true. If it were true, then, of course, it would be impossible to distinguish between (1) and (2), and it would follow that, since (1) certainly is true, what I am calling the dogma of internal relations is true too. But I imagine that Mr. Russell himself would now be willing to admit that, so far from being true, the statement that “ q can be deduced from p ” means the same as “ $p \supset q$ ” is simply an enormous “howler”; and I do not think I need spend any time in trying to show that it is so.

But it may be held that, though “ $p \text{ ent } q$ ” does not mean the same as “ $p \supset q$,” yet nevertheless from $(x) : \phi x . \psi x$, the proposition $(x) : \phi x . \text{ent} . \psi x$ does follow, for a somewhat more subtle reason; and, if this were so, it would again follow that what I am calling the dogma of internal relations must be true. It may be held, namely, that though $\phi A . \text{ent} . \psi A$ does not mean simply $\phi A \supset \psi A$, yet what it does mean is simply the conjunction “ $\phi A \supset \psi A$, and this proposition is an instance of a true formal implication” (the phrase “formal implication” being understood in Mr. Russell’s sense, in which $(x) : \phi x . \psi x$ asserts a formal implication). This view as to what $\phi A . \text{ent} . \psi A$ means has, for instance, if I understand him rightly, been asserted by Mr. O. Strachey in *Mind*, N.S., 93; since he asserts that, in his opinion, this is what Professor C. I. Lewis means by “ ϕA strictly implies ψA ,” and undoubtedly what Professor Lewis means by this is what I mean by $\phi A . \text{ent} . \psi A$. And the same view has been frequently suggested (though I do not know that he has actually asserted it) by Mr. Russell himself (*e.g.*, *Principia Mathematica*, p. 21). If this view were true, then, though $(x) : \phi x . \text{ent} . \psi x$ would not be identical in meaning with $(x) : \phi x . \psi x$, yet it would follow from it; since, if

$$(x) : \phi x . \text{ent} . \psi x$$

were true, then every particular assertion of the form $\phi A \supset \psi A$, [59] would not only be true, but would be an instance of a true formal implication (namely $(x) : \phi x . \psi x$) and this, according to the proposed definition, is all that $(x) : \phi x . \text{ent} . \psi x$ asserts. If, therefore, it were true, it would again follow that all relational properties must be internal. But that this view also is untrue appears to me perfectly obvious. The proposition that I am in this room does materially imply that I am more than five years old, since both are true; and the assertion that it does is also an instance of a true formal implication, since it is in fact true that all the persons in this room are more than five years old; but nothing appears to me more obvious than [sic] that the second of these two

propositions can *not* be deduced from the first—that the kind of relation which holds between the premisses, and conclusion of a syllogism in *Barbara* does *not* hold between them. To put it in another way: it seems to me quite obvious that the properties “being a person in this room” and “being more than five years old” are not related in the kind of way in which “being a right angle” is related to “being an angle,” and which we express by saying that, in the case of every term, the proposition that that term is an angle can be deduced from the proposition that it is a right angle.

These are the only two suggestions as to the meaning of “ p ent q ” known to me, which, if true, would yield the result that (2) does follow from (1), and that therefore all relational properties are internal; and both of these, it seems to me, are obviously false. All other suggested meanings, so far as I know would leave it true that (2) does not follow from (1), and therefore that I may possibly be right in maintaining that some relational properties are external. It might, for instance, be suggested that the last proposed definition should be amended as follows: that we should say: “ p ent q ” means “ p) q ,” and this proposition is an instance of a formal implication, which is not merely true but *self-evident*, like the laws of Formal Logic.” This proposed definition would avoid the paradoxes involved in Mr. Strachey’s definition, since such true formal implications as [60] “All the persons in this room are more than five years old” are certainly not self-evident; and, so far as I can see, it may state something which is in fact true of p and q , whenever and only when p ent q . I do not myself think that it gives the meaning of “ p ent q ,” since the kind of relation which I see to hold between the premisses and conclusion of a syllogism seems to me to be one which is purely “objective” in the sense that no psychological term, such as is involved in the meaning of “self-evident,” is involved in its definition (if it has one). I am not, however, concerned to dispute that some such definition of “ p ent q ” as this may be true. Since it is evident that, even if it were, my proposition that $(x) : \phi x . \text{ent} . \psi x$ does not follow from $(x) : \phi x .) . \psi x$, would still be true; and hence also my contention that (2) does not follow from (1).

So much by way of arguing that we are not bound to hold that all relational properties are internal in the particular sense, with which we are now concerned, in which to say that they are means that in every case in which a thing A has a relational property, it follows from the proposition that a term has *not* got that property that the term in question is *other* than A. But I have gone further and asserted that some relational properties certainly are *not* internal. And in defence of this proposition I do not know that I have anything to say but that it seems to me evident in many cases that a term which *has* a certain relational property *might* quite well not have had it: that, for instance, from the mere proposition that this is this, it by no means follows that this has to other things all the relations which it in fact has. Everybody, of course, must admit that if all the propositions which assert of it that it has these properties, do in fact follow from the proposition that this is this, we cannot see that they do. And so far as I can see, there is no reason of any kind for asserting that they do, except the confusion which I have exposed. But it seems to me further that we can see in many cases that the proposition that this has that relation does *not*

follow from the fact that it [61] is this: that, for instance, the proposition that Edward VII was father of George V *is a mere* matter of fact.

I want now to return for a moment to that other meaning of “internal,” in which to say that ϕ is internal to A means not merely that anything which had not ϕ would necessarily be *other* than A, but that it would necessarily be *qualitatively* different. I said that this was the meaning of “internal,” in which the dogma of internal relations holds that all relational properties are “internal”; and that one of the most important consequences which followed from it, was that all relational properties are “internal” in the less extreme sense that we have just been considering. But, if I am not mistaken, there is another important consequence which also follows from it, namely, the Identity of Indiscernibles. For if it be true in the case of every relational property that any term which had not that property would necessarily be qualitatively different from any which had, it follows of course that, in the case of any two terms one of which has a relational property which the other has not, the two are qualitatively different. But, from the proposition that x is other than y , it does follow that x has some relational property which y has not; and hence, if the dogma of internal relations be true, it will follow that if x is other than y , x is always also qualitatively different from y , which is the principle of Identity of Indiscernibles. This is, of course, a further objection to the dogma of internal relations, since I think it is obvious that the principle of Identity of Indiscernibles is not true. Indeed, so far as I can see, the dogma of internal relations essentially consists in the joint assertion of two indefensible propositions: (1) the proposition that, in the case of no relational property, is it true of any term which has got that property that it might not have had it and (2) the Identity of Indiscernibles.

I want, finally, to say something about the phrase which Mr. Russell uses in the *Philosophical Essays* to express the dogma of internal relations. He says it may be expressed in [62] the form “Every relation is grounded in the natures of the related terms” (p. 160). And it can be easily seen, if the account which I have given be true, in what precise sense it does hold this. Mr. Russell is uncertain as to whether by the nature of a term is to be understood the term itself or something else. For my part it seems to me that by a term’s nature is meant, not the term itself, but what may roughly be called all its qualities as distinguished from its relational properties. But whichever meaning we take, it will follow from what I have said, that the dogma of internal relations does imply that every relational property which a term has is, in a perfectly precise sense *grounded* in its nature. It will follow that every such property is *grounded* in *the term*, in the sense that, in the case of every such property, it *follows* from the mere proposition that that term is that term that it has the property in question. And it will also follow that any such property is grounded in the qualities which the term has, in the sense, that if you take *all* the qualities which the term has it will again follow in the case of each relational property, from the proposition that the term has *all* those qualities, that it has the relational property in question; since this is implied by the proposition that in the case of any such property, any term which had not had it would necessarily have been different in quality from the term in question. In both of these two senses,

then, the dogma of internal relations does, I think, imply that every relational property is grounded in the nature of every term which possesses it: and in this sense that proposition is false. Yet it is worth noting, I think, that there is another sense of “grounded” in which it may quite well be true that every relational property is grounded in the nature of any term which possesses it. Namely that, in the case of every such property, the term in question has some quality *without* which it could not have had the property. In other words that the relational property *entails* some quality in the term, though no quality in the term *entails* the relational property.