Answer the following questions in the spaces below them.

1. (7 points each) Give the substitution instance using the constant ‘a’ for each of the following sentences of PL:

   a. \((\exists x)(Rxa \land (\exists y)(Byx \lor (\exists z)Czxa))\)

      \(\text{Raa} \land (\exists y)(\text{Bya} \lor (\exists z)\text{Czaa})\)

   b. \((\forall w)(\forall y)(Vybc \supset (\exists x)(Pyxw \land Pwwa))\)

      \((\forall y)(Vybc \supset (\exists x)(Pyxa \land Paaa))\)

2. (9 points) Show all the subformulas of the following PL sentence:

\(~(\forall x)(\forall y)((\exists z)\sim Pxz \equiv \sim (Ryc \supset Cxya))\)

\(~(\forall x)(\forall y)((\exists z)\sim Pxz \equiv \sim (Ryc \supset Cxya))\)

\((\forall x)(\forall y)((\exists z)\sim Pxz \equiv \sim (Ryc \supset Cxya))\)

\((\forall y)((\exists z)\sim Pxz \equiv \sim (Ryc \supset Cxya))\)

\((\exists z)\sim Pxz \equiv \sim (Ryc \supset Cxya)\)

\((\exists z)\sim Pxz\)

\(~(Ryc \supset Cxya)\)

\(Ryc \supset Cxya\)

\(Ryc\)

\(Cxya\)
3. (7 points each) Symbolize the following sentences in PL1, using the symbolization key provided.

UD: Everything
s: the U.N. Security Council u: The United States i: Iraq
Mxy: x is a member of y Cx: x is a country b: Britain
Vxyz: x would vote in y to invade z Px: x is a person

a. If any country would vote in the U.N. Security Council to invade Iraq, the United States would.

$$(\exists x)V_xsi \supset Vusi$$

b. Iraq is not a member of the U.N. Security Council, and so it would not vote in the Security Council to invade itself.

$$\sim Mis \& \sim Visi$$

c. Although Britain and the United States would vote in the U.N. Security Council to invade Iraq, no other member of the Security Council would.

$$(Vbsi \& Vusi) \& \sim(\exists x)(\sim(x = u \lor x = b) \& Vxsi)$$
4. (7 points each) Symbolize the following sentences in PLI, using the symbolization key provided.

UD: Positive integers (1, 2, 3, . . .)

f: four  Pxyz: z is the product of x and y
o: one   Lxy: x is less than y
t: two

a. The positive integer that is less than two is less than four.

(∃x)((Lxt & (∀y)(Lyt ⊃ x = y)) & Lxf)

b. If the product of two positive integers is the same as one of them, then at least one of the two positive integers is the positive integer one.

(∀x)(∀y)((Pxyx ∨ Pxyy) ⊃ (x = o ∨ y = o))

c. At least three positive integers are less than four.

(∃x)(∃y)(∃z)(((Lxf & Lyf) & Lzf) & ((∼ x = y & ∼ y = z) & ∼ x = z))
5. (7 points each) Symbolize the following sentences in PL, providing your own symbolization key.

Symbolization key

UD: Persons
Wx: x is a wild dreamer  Fx: x is eventually frustrated
Dx: x dares to dream  Rx: x will be rewarded with great success
Cx: x is a child  Hx: x wants to be Harry Potter’s friend

a. A wild dreamer is eventually frustrated.

(∀x)(Wx ⊃ Fx)

b. Only someone who dares to dream will be rewarded with great success.

(∀x)(Rx ⊃ Dx)

c. Children want to be Harry Potter’s friend, but they are eventually frustrated.

(∀x)(Cx ⊃ (Hx & Fx))

Note: Because Harry Potter does not exist, he is not in the UD and no constant can refer to him.
6. (7 points each) Give fluent readings of the following sentences of PLI, using the symbolization key provided.

UD: Everything

d: Gray Davis
Rx: $x$ is a Republican
Iy: $x$ increases its fees
Py: $x$ proposes to cut the budget of $y$
Px: $x$ administers prisons
Ax: $x$ is a state agency

a. $(\forall x)(Rx \supset (\forall y)((Ay \& \neg Iy) \supset Pxy))$

Republicans propose to cut the budget of all agencies that do not increase their fees.

b. $(\exists x)[(Ax \& \neg Pdx \& (\forall y)(Ay \& \neg Pdy) \supset x = y)] \& Px$

The state agency whose fees Gray Davis does not propose to cut administers prisons.