Second Midterm Solutions  
Philosophy 112  
Winter 2003

Please work all the problems in the space provided. Each problem is worth 20 points. You may use only the rule set noted on the individual problems, except that you may use the falsum rule on any problem. Please note that with PD+ and PDI, you are not required to use any of extra rules provided by those rule-sets, though in the case of PDI, the use of identity rules may be unavoidable.

1. Prove that the following is a theorem of PD.

\((\forall x)Fx \lor (\exists x)\sim Fx\)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>(\sim((\forall x)Fx \lor (\exists x)\sim Fx))</td>
</tr>
<tr>
<td>2</td>
<td>(\sim Fa)</td>
</tr>
<tr>
<td>3</td>
<td>((\exists x)\sim Fx)</td>
</tr>
<tr>
<td>4</td>
<td>((\forall x)Fx \lor (\exists x)\sim Fx)</td>
</tr>
<tr>
<td>5</td>
<td>(\sim((\forall x)Fx \lor (\exists x)\sim Fx))</td>
</tr>
<tr>
<td>6</td>
<td>(Fa)</td>
</tr>
<tr>
<td>7</td>
<td>((\forall x)Fx)</td>
</tr>
<tr>
<td>8</td>
<td>((\forall x)Fx \lor (\exists x)\sim Fx)</td>
</tr>
<tr>
<td>9</td>
<td>((\forall x)Fx) (\lor (\exists x)\sim Fx)</td>
</tr>
</tbody>
</table>
2. Prove the equivalence of the following two sentences of \( PD \).

\[
\sim(\forall x)Fx \\
(\exists y)\sim Fy
\]

\[\begin{array}{ll}
1 & \sim(\forall x)Fx \\
2 & \sim(\exists y)\sim Fy \\
3 & \sim Fa \\
4 & (\exists y)\sim Fy \quad 3 \exists I \\
5 & \sim(\exists y)\sim Fy \quad 2 \text{ R} \\
6 & Fa \quad 2-5 \sim E \\
7 & (\forall x)Fx \quad 6 \forall I \\
8 & \sim(\forall x)Fx \quad 1 \text{ R} \\
9 & (\exists y)\sim Fy \quad 1-8 \sim E
\end{array}\]

\[\begin{array}{ll}
1 & (\exists y)\sim Fy \\
2 & \sim Fa \\
3 & (\forall x)Fx \\
4 & Fa \quad 3 \forall E \\
5 & \sim Fa \quad 2 \text{ R} \\
6 & \sim(\forall x)Fx \quad 2-5 \sim I \\
7 & \sim(\forall x)Fx \quad 1 \ 2-6 \exists E
\end{array}\]
3. Prove that the following derivability relation holds in \( PD^+ \).

\[ \{ (\forall x)(Qx \lor Ixb) \} \vdash (\exists x)Qx \lor (\forall x)Ixb \]

<table>
<thead>
<tr>
<th></th>
<th>((\forall x)(Qx \lor Ixb))</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(\neg((\exists x)Qx \lor (\forall x)Ixb))</td>
<td>Assumption</td>
</tr>
<tr>
<td>3</td>
<td>(\neg(\exists x)Qx \land \neg(\forall x)Ixb)</td>
<td>2 DeM</td>
</tr>
<tr>
<td>4</td>
<td>(\neg(\exists x)Qx)</td>
<td>3 &amp;E</td>
</tr>
<tr>
<td>5</td>
<td>(\forall x \neg Qx)</td>
<td>4 QN</td>
</tr>
<tr>
<td>6</td>
<td>(\neg Qa)</td>
<td>5 &amp;E</td>
</tr>
<tr>
<td>7</td>
<td>(Qa \lor Iab)</td>
<td>1 &amp;E</td>
</tr>
<tr>
<td>8</td>
<td>(Iab)</td>
<td>6 7 DS</td>
</tr>
<tr>
<td>9</td>
<td>(\forall x)Ixb)</td>
<td>8 &amp; I</td>
</tr>
<tr>
<td>10</td>
<td>((\exists x)Qx \lor (\forall x)Ixb)</td>
<td>9 &amp; I</td>
</tr>
<tr>
<td>11</td>
<td>((\exists x)Qx \lor (\forall x)Ixb)</td>
<td>2-10 \neg E</td>
</tr>
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</table>
4. Show that the following argument is valid in PDI.

The current President of the United States was born in Connecticut. Connecticut and Texas are different, and nothing has more than one birthplace. Therefore, no current president of the United States was born in Texas.

\((\exists x)((P_xu \& (\forall y)(P_yu \supset x = y)) \& Bxc)\)

\(~c = t \& (\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z))\)

\begin{center}
\begin{tabular}{ | c | p{10cm} | }
\hline
1 & \((\exists x)((P_xu \& (\forall y)(P_yu \supset x = y)) \& Bxc)\) & Assumption \\
2 & \(~c = t \& (\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z))\) & Assumption \\
3 & \((\exists x)(P_xu \& Bxt)\) & Assumption \\
4 & \((Pau \& (\forall y)(P_yu \supset a = y)) \& Bac\) & Assumption \\
5 & Bac & 4 \& E \\
6 & Pau \& (\forall y)(P_yu \supset a = y) & 4 \& E \\
7 & (\forall y)(P_yu \supset a = y) & 6 \& E \\
8 & (\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z)) & 2 \& E \\
9 & (\forall y)(\forall z)(Bay \supset (Baz \supset y = z)) & 8 \forall E \\
10 & (\forall y)(Bac \supset (Baz \supset c = z)) & 9 \forall E \\
11 & Bac \supset (Bat \supset c = t) & 10 \forall E \\
12 & Bat \supset c = t & 5 11 11 \supset E \\
13 & Pru \& Brt & Assumption \\
14 & Pru & 13 \& E \\
15 & Brt & 13 \& E \\
16 & Pru \supset a = r & 7 \forall E \\
17 & a = r & 14 16 \supset E \\
18 & Bat & 15 17 = E \\
19 & c = t & 12 18 \supset E \\
20 & ~c = t & 2 \& E \\
21 & c = t \& ~c = t & 19 20 \& I \\
22 & c = t \& ~c = t & 3 13-21 \exists E \\
23 & c = t \& ~c = t & 1 4-22 \exists E \\
24 & c = t & 23 \& E \\
25 & ~c = t & 23 \& E \\
26 & \sim(\exists x)(P_xu \& Bxt) & 3-25 \sim E \\
\hline
\end{tabular}
\end{center}
5. Using a symbolization key that reveals as much logical structure as possible, symbolize the following sentences in PL. Then prove that the set of PL sentences is inconsistent in PD+.

Only an earlier event causes an event. One event is earlier than another only if the later event is not earlier than the earlier one. Some event causes every event.

UD: The set of all events
Exy: x is earlier than y
Cxy: x causes y

\[(\forall x)(\forall y)(Cxy \supset Exy)\]
\[(\forall x)(\forall y)(Exy \supset \sim Eyx)\]
\[(\exists x)(\forall y)Cxy\]

1. \[(\forall x)(\forall y)(Cxy \supset Exy)\] Assumption
2. \[(\forall x)(\forall y)(Exy \supset \sim Eyx)\] Assumption
3. \[(\exists x)(\forall y)Cxy\] Assumption
4. \[(\forall y)Cay\] Assumption
5. Caa 4 \forall E
6. \[(\forall y)(Cay \supset Eay)\] 1 \forall E
7. Caa \supset Eaa 6 \forall E
8. Eaa 5 7 \supset E
9. \[(\forall y)(Eay \supset \sim Eya)\] 2 \forall E
10. Eaa \supset \sim Eaa 8 \forall E
11. \sim Eaa 8 10 \supset E
12. \bot 8 11 \bot I
13. \bot 3 4-12 \exists E
14. \sim (\exists x)(\forall y)Cxy 13 \bot E