Object Language and Metalanguage

Formal Languages

- Sentence Logic and Predicate Logic are formal languages.
- A formal language is a set of sentences generated by rules of formation from a vocabulary.
- The sentences of Sentence Logic and Predicate Logic are not part of natural language (though some may resemble natural-language sentences).
- The formal languages Sentence Logic and Predicate Logic are the objects of our study, and as such they are called object languages.

The Metalanguage

- If we are going to state anything about an object language, we must make use of a language.
- We call a language used to study an object language a metalanguage.
- In theory, the metalanguage may be identical to or include the object language.
  - We use English to study English in linguistics.
- We will strictly separate our metalanguage (English with some extra technical vocabulary) from our object languages.
- Keeping the languages separate allows us to avoid semantical paradox (Tarski).

Use and Mention

- When we talk about an item of language, we are said to mention it.
- Whenever an item of any object language is mentioned, it must be placed within single quotation marks.
- We may use English to mention an item of English.
  - ‘Bush’ has four letters and starts with a ‘B’.
  - ‘George W. Bush was born in Texas’ is false.
  - ‘This sentence is false’ is true.
Metavariables

- We may also use English to mention items of Sentence Logic and Predicate Logic.
  - ‘⊃’ is a connective of Sentence Logic.
  - ‘P ⊃ Q’ is a conditional.
  - If ‘P’ is true and ‘P ⊃ Q’ is true, then ‘Q’ is true.
- To state general facts about Sentence Logic and Predicate Logic, we must use expressions that designate classes of items of the object language.
  - Such expressions are called metavariabes.

Metavariables for Sentences and Sets of Sentences

- To mention metavariables themselves, single quotation marks must be used.
- ‘Q’ through ‘Z’ will be used as metavariables for sentences of both Sentence Logic and Predicate Logic.
- ‘X’ through ‘Z’ will be used as metavariables for sets of sentences of both Sentence Logic and Predicate Logic.

Metavariables and Connectives

- In English, we refer to connectives of Sentence Logic using such expressions as ‘sign of negation’ or ‘sign of the conditional’.
- We may also refer to them by mentioning them:
  - ‘∼’
  - ‘⊃’
- We must have a way to combine our use of metavariables with reference to connectives.

Names of Themselves

- We could make general statements about connectives using English.
  - If X is a sentence of Sentence Logic, then the result of prefixing the sentence X refers to with a ‘∼’ (or sign of negation) and surrounding the result with parentheses is a sentence of Sentence Logic.
- This kind of statement is obviously very cumbersome.
- We want to say: ‘If X is a sentence, then (∼X) is a sentence’.
- Strictly speaking, this mixes the object language with the metalanguage.
- So we say (without paradox) that in ‘∼X’, ‘∼’ is used as a name of itself, or autonomously (Carnap).
Metavariables for Predicate Logic

- The vocabulary of Predicate Logic is an extension of the vocabulary of Sentence Logic.
- It contains all the expressions of the vocabulary of Sentence Logic along with other expressions proper to Predicate Logic.
- The metavariables for these new vocabulary items will be introduced when the vocabulary items themselves are introduced.

Sets

- An important part of the metalanguage for Sentence Logic and Predicate Logic is set-theoretic notation.
  - We have already described metavariables referring to sets of sentences.
- Sets are indicated by enclosure in curly brackets ‘{’ and ‘}’.
  - ‘{P, P ⊃ Q}’ indicates the set consisting of ‘P’ and ‘P ⊃ Q’.
- Ordered sets are indicated by enclosure in angle brackets ‘⟨’ and ‘⟩’. ‘⟨P, P ⊃ Q⟩’ indicates the set consisting of ‘P’ and ‘P ⊃ Q’ in that order: first ‘P’ and then ‘P ⊃ Q’.

Specifying Sets

- There are two methods of specifying the contents of a set.
- The method of enumeration simply lists the contents of the set using linguistic items that refer to the objects in the set.
  - {The President of the United States, Donald Rumsfeld, 2005}
  - ⟨Adam, Eve⟩
The method of description states what condition must be satisfied for something to be a member of the set.
  - {x: x is blond}: the set of all x such that x is blond, or the set of all blonds
  - {(x,y): x > y}: the set of all ordered pairs x and y such that x is greater than y