Please work all the problems in the space provided. Each problem is worth 20 points. You may use only the rule set noted on the individual problems, except that you may use the falsum rule on any problem.

1. Prove that the following is a theorem of PD. This is a symbolization of the barber paradox: no barber shaves all and only those who do not shave themselves.

\sim(\exists x)(\forall y)(Sxy \equiv \sim Syy)
2. Prove the equivalence of the following two sentences of \( PD \).

\[(\forall x)(\forall y) (Fx \supset Gy)\]
\[(\forall x)(Fx \supset (\forall y)Gy)\]
3. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Show that it is valid in $PD_+$. 

No number is smaller than itself. Hence, there is no number that is smaller than all numbers.
4. Show that the following argument is valid in PD+. (This symbolizes an argument from Plato’s *Meno*. Something can be taught only if there are teachers and pupils of it. Virtue has no teachers or pupils, so no virtue can be taught.)

\[
(\forall x)(Cx \supset ((\exists y)Tyx \land (\exists y)Pyx))
(\forall x)(Vx \supset \neg(\exists y)(Tyx \lor Pyx))
\]

\[
\neg(\exists x)(Vx \land Cx)
\]
5. Prove that the following set of sentences is inconsistent in \( PDI \).
\[
\{(\exists x)(Fx & (\forall y)(Fy \supset x = y)), \sim g = a, Fa, Fg\}
\]