Answer the following questions in the spaces below them.

1. (7 points each) Give the substitution instance using the constant ‘a’ for each of the following sentences of PL:
   a. $(\exists x)(Rxa \land (\exists y)(Bxy \lor (\exists z)Cza))$
   
   b. $(\forall w)(\forall y)(\forall z)(\exists x)(Pyxw \land Pwwa)$

2. (9 points) Show all the subformulas of the following PL sentence:
   \[ \neg(\forall x)(\forall y)((\exists z)\neg Pxz \equiv \neg(Ryc \supset Cxy)) \]
3. (7 points each) Symbolize the following sentences in PLI, using the symbolization key provided.

UD: Everything
s: the U.N. Security Council u: The United States i: Iraq
Mxy: x is a member of y Cx: x is a country b: Britain
Vxyz: x would vote in y to invade z Px: x is a person

a. If any country would vote in the U.N. Security Council to invade Iraq, the United States would.

b. Iraq is not a member of the U.N. Security Council, and so it would not vote in the Security Council to invade itself.

c. Although Britain and the United States would vote in the U.N. Security Council to invade Iraq, no other member of the Security Council would.
4. (7 points each) Symbolize the following sentences in PLI, using the symbolization key provided.

**UD:** Positive integers (1, 2, 3, . . .)

- **f:** four
- **o:** one
- **t:** two

- **Pxyz:** z is the product of x and y
- **Lxy:** x is less than y

a. The positive integer that is less than two is less than four.

b. If the product of two positive integers is the same as one of them, then at least one of the two positive integers is the positive integer one.

c. At least three positive integers are less than four.
5. (7 points each) Symbolize the following sentences in PL, providing your own symbolization key.

Symbolization key

a. A wild dreamer is eventually frustrated.

b. Only someone who dares to dream will be rewarded with great success.

c. Children want to be Harry Potter’s friend, but they are eventually frustrated.
6. (7 points each) Give fluent readings of the following sentences of PLI, using the symbolization key provided.

UD: Everything

d: Gray Davis

x: increases its fees

P: x administers prisons

Ax: x is a state agency

Rx: x is a Republican

Pxy: x proposes to cut the budget of y

a. \((\forall x)(Rx \supset (\forall y)((Ay \& \sim Iy) \supset Pxy))\)

b. \((\exists x)((Ax \& \sim Pdx \& (\forall y)(Ay \& \sim Pdy) \supset x = y)) \& Px] \)