Solutions to Selected Exercises Using Formal Semantics

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2-5 a6)
To solve this problem, we must appeal to our general knowledge. I know of at least one unhappy U.S. citizen over 21, Kyle, who is not a millionaire. Let \( d \) be an arbitrary variable assignment. \((\text{Kyle}) \notin v(M)\), and \((\text{Kyle}) \notin v(H)\), so \( (d[\text{Kyle}/x](x)) \) is not in either \( v(M) \) or \( v(H) \). So, \( d[\text{Kyle}/x] \) does not satisfy ‘Hx’ or ‘Mx’. So \( d[\text{Kyle}/x] \) satisfies ‘\( \neg Hx \)’ and ‘\( \neg Mx \)’. Therefore, \( d[\text{Kyle}/x] \) satisfies ‘\( \neg Hx \land \neg Mx \)’. Thus, \( d[\text{Kyle}/x] \) satisfies ‘\( (Hx \land Mx) \lor (\neg Hx \land \neg Mx) \)’. So \( d \) satisfies ‘\( (\exists x)[(Hx \land Mx) \lor (\neg Hx \land \neg Mx)] \)’. Since \( d \) is arbitrary, the sentence is satisfied by all variable assignments, and the sentence is true in the interpretation.

2-5 a8)
I know a number of U.S. citizens over 21 who are happy but not millionaires. Josh is one of them. \((\text{Josh}) \notin v(M)\), so for arbitrary variable assignment \( d \), \( (d[\text{Josh}/x](x)) \notin v(M) \), in which case \( d[\text{Josh}/x] \) does not satisfy ‘Mx’. But \((\text{Josh}) \in v(H)\), and so \( (d[\text{Josh}/x](x)) \in v(H) \) and \( d[\text{Josh}/x] \) satisfies ‘Hx’. Then \( d[\text{Josh}/x] \) does not satisfy ‘\( Hx \supset Mx \)’. Therefore, \( d \) does not satisfy ‘\( (\forall x)[Hx \supset Mx] \)’. It follows that \( d \) satisfies ‘\( (\forall x)[Hx \supset Mx] \supset (\exists x)\neg Mx \)’. Since \( d \) is arbitrary, all variable assignments satisfy the sentence, and the sentence is true in the given interpretation.

2-5 b7)
The number 5 is odd but is not greater than or equal to 17. Since \( (5) \in v(O) \), and hence \( (d[5/x](x)) \in v(O) \), \( d[5/x] \) satisfies ‘Ox’. Further, since \( (5, 18) \notin v(K) \), so neither is \( (d[5/x](x), v(a_{17})) \) a member of \( v(K) \). Hence \( d[5/x] \) does not satisfy ‘\( Kxa_{17} \)’, in which case it does not satisfy ‘\( \neg Kxa_{18} \land Kxa_{17} \)’. Therefore, \( d[5/x] \) does not satisfy ‘\( Ox \equiv (\neg Kxa_{18} \land Kxa_{17}) \)’. Since there is at least one \( x \)-variant of \( d \) whose value is a member of the domain that does not satisfy the open sentence, the universally quantified sentence ‘\( (\forall x)[Ox \equiv (\neg Kxa_{18} \land Kxa_{17})] \)’ is false in the interpretation.
Let $I$ be an interpretation which makes ‘$(\forall x)(Bx \land Lxe)$’ true. Then for all variable assignments $d$ based on $I$, the sentence is satisfied. This holds just in case for all members $u$ of the domain of $I$, $d[u/x]$ satisfies ‘$Bx \land Lxe$’. Thus, $d[u/x]$ satisfies both ‘$Bx$’ and ‘$Lxe$’, in which case it satisfies ‘$Bx$’. Then $d$ satisfies ‘$(\forall x)Bx$’, and since this holds for all variable assignments based on $I$, the sentence is true in $I$. So given that the premise is true in an interpretation, so is the conclusion, and the argument is valid.