Exercises for Functions and Identity

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1 Exercises FI-1

State whether or not the following expressions are sentences of Predicate Logic. If not, state why not.

a) \( f(a) \)
b) \( f(a) = \sim b \)
c) \( \sim f(a) = a \)
d) \( Lf(b)f(a) \)
e) \( Lf(a) \)
f) \( Babcdef(a) \)
g) \( Bf(a) = b \)
h) \( f(f(f(a))) = f(a) \)

2 Exercises FI-2

Given the following transcription guide, determine whether the following sentences are true or false.

\( D \) = the set of all positive integers
\( o \): 1
\( w \): 2
\( t \): 3
\( f \): the addition function
\( s \): the successor function [e.g., the successor of 1 is 2]
\( Gxy \): \( x \) is greater than \( y \)

a) \( Gws(o) \)
b) \( f(o,w) = t \)
c) \( s(s(o)) = s(w) \)
d) \( s(w) = f(o,w) \)
e) \( Gf(s(o),s(o))w \)
3 Exercises FI-3

Unusing the above transcription guide, transcribe the following sentences into Predicate Logic.

a) $1 + 3 > 1 + 2$

b) The successor of 1 is not greater than the successor of 2.

c) The successor of 1 is greater than or equal to 1.

4 Exercises FI-4

Show that the following derivability claims hold, using Sentence Logic rules plus the introduction and elimination rules for ‘=’.

a) $\{a = b, b = c\} \vdash a = c$

b) $\{a = b\} \vdash Ka \equiv Kb$

c) $\{Lab, b = f(a), c = f(b), Lf(b)c\} \vdash La f(a) & Lcc$

d) $\{\sim a = c\} \vdash \sim a = b \lor \sim b = c$ [more challenging]

5 Exercises FI-5

Show that the following property [“Substitutivity of Identicals”] holds in the semantics for Predicate Logic. Assume the antecedent of the conditional and give an argument for why the consequent follows.

If ‘Lea’ and ‘a = e’ are true in an interpretation, then ‘Lee’ and ‘Laa’ are true in that interpretation.