Natural Deduction Rules with Iterated Quantifiers

Iterated Universal Quantifiers

• There are no restrictions on instantiating universal sentences, which means there is no problem with iterated universal quantifiers.
• For example, \((\forall x)(\forall y)Fxy\) can be instantiated as:
  – \((\forall y)Fay\), which can be instantiated as:
    * \(Faa\), or
    * \(Fab\), etc.

Iterated Existential Quantifiers

• The instantiation of existential quantifiers is subject to restrictions which rule out some instantiations in the case of multiple quantifiers.
• For example, the following pattern of instantiation is not permitted:

\[
\begin{array}{c|l}
1 & (\exists x)(\exists y)Fxy & P \\
2 & \text{a} (\exists y)Fay & A \\
3 & \text{a} Faa & A \\
4 & & \\
\end{array}
\]

• The second instantiation violates the restriction that the instantiating name be isolated in the derivation.

Hard Problems

• On page 94, a derivation in 21 steps is given.
• We are told what the two main strategic moves are, but not why they were chosen.
• Reductio is chosen as the basic strategy.
  – The reason is that it would be impossible to use \(\exists I\) to get the conclusion.
  – This condition usually holds, and so reductio is a good strategy for deriving existential sentences.
• The sub-strategy is to use reductio again to get ‘På’ in order to use \(\forall I\).
  – Once again, it is impossible to get this result in any other way.
  – This condition usually holds, so reductio is a good strategy for deriving atomic sentences that cannot be derived using instantiation.