Transcription and Restricted Quantifiers

Differences Among Quantifier Expressions

- Some English quantifier expressions are neutral with respect to the range of objects whose quantity they express.
  - Any, every, all, whatever - some
  - Anything, everything - something
  - There is, at least one is
- Other quantifier expressions apply only to a limited range of objects.
  - Anyone, anybody - someone, somebody (persons)
  - Anywhere, everywhere - somewhere (places)
  - Whenever, always when - sometimes (times)

Limiting the Domain

- One way of transcribing sentences with restricted quantifiers is to limit the domain to the objects to which the quantifiers are supposed to apply.
  - \( D = x: x \) is a person, so \( (\forall x) \) and \( (\exists x) \) apply only to persons.
  - ‘Everybody is happy’ is transcribed as \( (\forall x)Hx \), where \( Hx: x \) is happy.
- However, this only allows us to talk about items in the domain, so that in the last example, we could not transcribe ‘Everybody is happy sometimes’.

Restricted Quantifiers

- One way to transcribe sentences with a mixture of types of quantifier expressions is to create a new kind of quantifier in Predicate Logic: the restricted quantifier.
- We choose a predicate letter to symbolize the restricted range of objects.
  - ‘P’ stands for the set of all persons.
  - We write \( (\forall x)p \) for ‘everyone’, and \( (\exists x)p \) for ‘someone’.
  - ‘Everyone is happy’ is transcribed as \( (\forall x)pHx \).

We can mix restricted quantifiers to symbolize sentences containing more than one limited quantifier expression.

- ‘T’ stands for the set of all times.
- ‘Everyone is happy sometimes’ is transcribed as \( (\forall x)p(\exists y)_T Hxy \), where \( Hxy: x \) is happy at \( y \).
Semantics for Restricted Quantifiers

- Tarski-style semantics can be used to specify satisfaction-conditions for sentences with restricted quantifiers.
- For each restriction represented by a one-place predicate $S$, we generate the set $r(S)$ from $v(S)$ by stripping off the angle brackets.
  - Let $v(P) = \{ \langle \text{Adam} \rangle, \langle \text{Eve} \rangle \}$; $r(P) = \{ \text{Adam}, \text{Eve} \}$.
- Then we say that $d$ satisfies $(\exists u)_{S} P(u)$ if and only if for some object $o \in r(S)$, $d[o/u]$ satisfies $P(u)$.
  - Let $v(B) = \{ \langle \text{Adam} \rangle \}$; then $d[\text{Adam}/x]$ satisfies ‘$Bx$’, so $d$ satisfies ‘$(\exists x)_p Bx$’.
- Similarly, $d$ satisfies $(\forall u)_{S} P(u)$ if and only if for all objects $o \in r(S)$, $d[o/u]$ satisfies $P(u)$.

Eliminating Restricted Quantifiers

- Restricted quantifiers can be eliminated in favor of other constructions without change in truth-value.
- $(\exists u)_{S} P(u)$ is equivalent to $(\exists u)(S(u) & P(u))$.
  - ‘$(\exists x)_p Bx$’ is equivalent to ‘$(\exists x)(Px \& Bx)$’.
- $(\forall u)_{S} P(u)$ is equivalent to $(\forall u)(S(u) \supset P(u))$.
  - ‘$(\forall x)_p Bx$’ is equivalent to ‘$(\forall x)(Px \supset Bx)$’.
- The replacement of one form for the other (when authorized) may occur in an internal part of a sentence.
  - ‘$(\exists x)_p Lxe \supset (\exists x)_p Lex$’ is equivalent to ‘$(\exists x)(Px \& Lxe) \supset (\exists x)(Px \& Lex)$’.

Proof of Equivalence for Restricted Existentials

- $(\exists u)_{S} P(u)$ is true in $I$ if and only if (iff) it is satisfied by all variable assignments $d$ based on $I$.
- Let $I$ be an arbitrary interpretation and $d$ an arbitrary variable assignment based on $I$.
- $d$ satisfies $(\exists u)_{S} P(u)$ iff some object $o \in r(S)$, $d[o/u]$ satisfies $P(u)$.
  - iff for some object $o$ in $D$, $d[o/u]$ satisfies $P(u)$, and the one-tuple $(o) \in v(S)$ [by the definition of $r(S)$],
  - iff for some object $o \in D$, $d[o/u]$ satisfies $P(u)$ and $d[o/u]$ satisfies $S(u)$,
• iff for some object $o$ in $D$, $d[o/u]$ satisfies $S(u)$ & $P(u)$,
• iff $d$ satisfies $(\exists u)(S(u) \& P(u))$,
• Since $d$ is arbitrary, $(\exists u)S(u)$ is true in $I$ iff $(\exists u)(S(u) \& P(u))$ is true in $I$, QED.

Proof of Equivalence for Restricted Universals

• To save space, only the core of the proof is presented; the other steps are trivial.
• $d$ satisfies $(\forall u)S(u)P(u)$ iff for all objects $o \in r(S)$, $d[o/u]$ satisfies $P(u)$,
• iff for all objects $o \in D$, if $o \in r(S)$, then $d[o/u]$ satisfies $P(u)$,
• iff for all objects $o \in D$, if the one-tuple $\langle o \rangle \in v(S)$, then $d[o/u]$ satisfies $P(u)$,
• iff for all objects $o \in D$, if $d[o/u]$ satisfies $S(u)$, then $d[o/u]$ satisfies $P(u)$,
• iff for all objects $o \in D$, $d[o/u]$ satisfies $S(u) \supset P(u)$,
• iff, $d$ satisfies $(\forall x)(S(u) \supset P(u))$, QED.

Negated Restricted Quantifiers

• The following two logical equivalences hold, with a proof of the first below.
  - $\sim(\forall u)S(u)P(u)$ and $(\exists u)S(\sim P(u))$
  - $\sim(\exists u)S(u)$ and $(\forall u)\sim P(u)$
• $d$ satisfies $\sim(\forall u)S(u)P(u)$ iff $d$ does not satisfy $(\forall u)S(u)P(u)$,
• iff it is not the case that for all $o \in D$, if $o \in r(S)$, then $d[o/u]$ satisfies $P(u)$,
• iff for some $o \in D$, it is not the case that if $o \in r(S)$, then $d[o/u]$ satisfies $P(u)$,
• iff for some $o \in D$, $o \in r(S)$ and $d[o/u]$ does not satisfy $P(u)$,
• iff for some $o \in D$, $o \in r(S)$, and $d[o/u]$ satisfies $\sim P(u)$,
• iff $d$ satisfies $(\exists u)S(\sim P(u))$, QED.