Final Examination Solutions
Philosophy 112
Winter 2002

Please work all the problems in the space provided. All problems are weighted equally. You may use only the rule set noted on the individual problems. Please be sure that you do everything that is asked for in each problem.

1. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Show that it is valid in *PD* (which requires a derivation).

A person is humble if and only if he or she doesn't admire himself or herself. It follows that nobody who admires all humble people is humble.

UD: Persons
Hx: x is humble
Axy: x admires y

$(\forall x)(Hx \equiv \sim Axx)$

$\overline{\sim(\exists x)((\forall y)(Hy \supset Axy) \,\&\, Hx)}$

1	$(\forall x)(Hx \equiv \sim Axx)$	Assumption
2	$\quad (\exists x)((\forall y)(Hy \supset Axy) \,\&\, Hx)$	Assumption
3	$\quad\quad (\forall y)(Hy \supset Acy) \,\&\, Hc$	Assumption
4	$\quad\quad (Hc \supset Acc) \,\&\, Hc$	3 $\forall$ E
5	$\quad\quad Hc \supset Acc$	4 & E
6	$\quad\quad Hc$	4 & E
7	$\quad\quad Acc$	5 6 $\supset$ E
8	$\quad\quad Hc \equiv \sim Acc$	1 $\forall$ E
9	$\quad\quad \sim Acc$	7 8 $\equiv$ E
10	$\quad\quad \perp$	7 9 $\perp$ I
11	$\quad \perp$	2 3-10 $\exists$ E
12	$\quad \sim(\exists x)((\forall y)(Hy \supset Axy) \,\&\, Hx)$	11 $\perp$ E
13	$\sim(\exists x)((\forall y)(Hy \supset Axy) \,\&\, Hx)$	2-12 $\sim$ I

2. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Using any *semantical* technique for *PL* (not a derivation), determine whether it is quantificationally valid or invalid and defend your answer.

Nothing is larger than itself. So, nothing is larger than everything.

UD: Everything
Lxy: x is larger than y

∼(∃x)Lxx

∼(∃x)(∀y)Lxy

Suppose that for some arbitrary interpretation **I**, '∼(∃x)Lxx' is true on **I**. Then '(∃x)Lxx' is false on **I**. So no member of the UD satisfies the condition specified by 'Lxx'. No ordered pair of one object and itself is in the extension of 'L'. Now suppose '(∃x)(∀y)Lxy' is true on **I**. Then there is some member of the UD which satisfies the condition specified by '(∀y)Lxy'. This means that the member of the UD just mentioned bears the relation symbolized by 'L' to all members of the UD. Then it bears that relation to itself. But it has been shown that this is not the case. Therefore, '(∃x)(∀y)Lxy' is false on **I**, and '∼(∃x)(∀y)Lxy' is true on **I**, which was to be proved.

3. Using the definitions from the formal semantics, show that the following two sentences of *PL* are quantificationally equivalent.

(∀x)Gx
∼(∃x)∼Gx

Let **I** be an arbitrary interpretation. '(∀x)Gx' is true on **I** if and only if it is satisfied by all variable-assignments **d** for **I**. This holds if and only if 'Gx' is satisfied by all x-variants **d**[**u**/x] of all variable-assignments **d**. And this holds if and only if '∼Gx' is not satisfied by all x-variant **d**[**u**/x] of all variable-assignments **d**. This holds if and only if '(∃x)∼Gx' is not satisifed by all variable assignments **d**. And this holds if and only if '∼(∃x)∼Gx' is satisified by all variable assignments **d**, which in turn holds if and only if the sentence is true on **I**. Therefore, '(∀x)Gx' is true on **I** if and only if '∼(∃x)∼Gx' is true on **I**, for arbitrary **I**, which was to be proved.

4. Show that the following sentence is quantificationally indeterminate by constructing an interpretation on which it is true and one on which it is false. State why it is true on the interpretation on which it is true, and why it is false on the interpretation on which it is false.

(∀x)(∀y)(∃z)(x=z & y=z)

For an interprtation **I** on which the sentence is true, let the UD of **I** consist of a single object, the number 1. As with any object, 1 is identical with itself. Also, there is only one variable-assignment **d**, since each variable can be assigned only the number 1. Thus **d** satisfies 'x=z', and 'y = z', and so it satisifies 'x=z & y=z'. Since this holds for some z-variant of **d** (i. e., **d** itself), **d** satisifies '(∃z)(x=z & y=z)'. Further, **d** satisfies '(∀y)(∃z)(x=z & y=z)', since all its y-variants (which include only **d** itself) satisfy the sub-formula. For the same reason, all x-variants of **d** satisfy '(∀y)(∃z)(x=z & y=z)', in which case **d** satisifies '(∀x)(∀y)(∃z)(x=z & y=z)'. Then the sentence is true on **I**.

For an interpretation on which the sentence is false, we must use a domain with at least two objects. Let the UD = {1, 2}. For any variable-assignment **d**, **d**(x)=1, or **d**(x)=2. Now suppose **d**(x)=1. **d**[1/x, 2/y, 2/z] does not satisfy 'x = z,' Therefore, it does not satisfy 'x = z & y = z'. **d**[1/x, 2/y, 1/z] does not satisfy 'y = z', and so it does not satisfy 'x = z & y = z'. So , as these are the only two z-variants, no z-variant of **d**[1/x, 2/y] satisfies 'x = z & y = z'. So **d**[1/x, 2/y] does not satisfy '(∃x)(x = z & y = z)'. Therefore, **d**[1/x] does not satisfy '(∀y)(∃z)(x = z & y = z)'. So not all x-variants of **d** satisfy '(∀y)(∃z)(x = z & y = z)', in which case **d** does not satisfy '(∀x)(∀y)(∃z)(x = z & y = z)'. Then the sentence is false on **I**.

5. Show that the following set of sentences is quantificationally consistent by constructing an appropriate expanded truth-table.

$\{(\forall x)(Fax \lor (\exists y)Fya), \sim Faa\}$

N/A

6. Using the formal semantics for *PL*, determine the truth-value of the following sentence on the interpretation given. Show in detail how the truth-value is determined.

(∀x)(Ox ⊃ (∃y)Lyx)

UD: {1,2}
O: {<**u**>: **u** is odd}
L: {<$\mathbf{u}_1$,$\mathbf{u}_2$ >: $\mathbf{u}_1$ is less than $\mathbf{u}_2$}

The sentence is false on this interpretation.
The number one is odd, so <1> is in **I**(O). Thus **d**[1/x] satisfies ‘Ox’. No member of the UD is less than 1. Therefore, no ordered pair <**u**,1> is in **I**(L). So there is no y-variant of **d**[1/x] which satisfies ‘Lyx’. Hence, **d**[1/x] does not satisfy ‘(∃x)Lyx’. Then **d**[1/x] does not satisfy ‘Ox ⊃ (∃y)Lyx’. Because at least one x-variant of **d** does not satisfy ‘Ox ⊃ (∃y)Lyx’, **d** itself does not satisfy ‘(∀x)(Ox ⊃ (∃y)Lyx)’. And so the sentence is false on this interpretation.