Final Examination
Philosophy 112
Winter 2003

Please work all the problems in the space provided. You may use only the techniques noted on the individual problems. Please be sure that you do everything that is asked for in each problem. Also, in each answer, bring out as much detail as possible.

Name________________________________

1. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key, *with a UD that includes everything.* Show that it is valid in *PD* (which requires a derivation). (15 points)

Some people are vegetarians, and vegetarians do not eat meat. Therefore, some people do not eat meat.

UD: Everything
Px: x is a person
Vx: x is a vegetarian
Mx: x eats meat

$$\frac{(\exists x)(Px \,\&\, Vx) \,\&\, (\forall x)(Vx \supset {\sim}Mx)}{(\exists x)(Px \,\&\, {\sim}Mx)}$$

1	$(\exists x)(Px \,\&\, Vx) \,\&\, (\forall x)(Vx \supset {\sim}Mx)$	Assumption
2	$(\exists x)(Px \,\&\, Vx)$	1 &E
3	$\quad$ $Pa \,\&\, Va$	Assumption
4	$\quad$ $(\forall x)(Vx \supset {\sim}Mx)$	1 &E
5	$\quad$ $Va \supset {\sim}Ma$	4 ∀E
6	$\quad$ Va	3 &E
7	$\quad$ ${\sim}Ma$	5 6 ⊃E
8	$\quad$ Pa	3 &E
9	$\quad$ $Pa \,\&\, {\sim}Ma$	7 8 &I
10	$\quad$ $(\exists x)(Px \,\&\, {\sim}Mx)$	9 ∃I
11	$(\exists x)(Px \,\&\, {\sim}Mx)$	2 3-10 ∃E

Name______________________________

2. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Using any *semantical* technique for *PL* (*not a derivation*), determine whether it is quantificationally valid or invalid and defend your answer. (15 points)

If everyone is going to the mall, then Jason is going.

If anyone is going to the mall, then Jason is going.

UD: Everything
m: the mall
j: Jason
Px: x is a person
Gxy: x is going to y

$$\frac{(\forall x)(Px \supset Gxm) \supset Gjm}{(\exists x)(Px \ \& \ Gxm) \supset Gjm}$$

The argument is quantificationally invalid. Consider an interpretation on which one person is going to the mall and Jason is not. There is a member of the UD (Jason) who satisfies the condition specified by ‘Px’ but does not specify the condition specified by ‘Gxm’. Hence, at least one member of the UD does not satisfy the conditional ‘Px ⊃ Gxm’. So the sentence ‘(∀x)(Px ⊃ Gxm)’ is false on the interpretation. That makes the larger conditional ‘(∀x)(Px ⊃ Gxm) ⊃ Gjm’ is true on the interpretation. The consequent of the conclusion is false because Jason is not going to the mall. The antecedent, however, is true. One member of the UD satisfies ‘Px & Gxm’, and so the existential sentence ‘(∃x)(Px & Gxm)’ is true. This makes the conclusion of the argument false. Since there is an interpretation on which the premise is true and the conclusion is false, the argument is invalid.

Name______________________________

3. Using the definitions from the formal semantics, show that the following two sentences of *PL* are quantificationally equivalent. (15 points)

(∃y)Fy ∨ (∃x)Gx
(∃y)(Fy ∨ (∃x)Gx)

Let **I** be an arbitrary interpretation and let **d** be a variable assignment based on **I**. Suppose '(∃y)Fy ∨ (∃x)Gx' is true on **I**. Then **d** satisfies '(∃y)Fy ∨ (∃x)Gx'. Therefore, **d** satisfies '(∃y)Fy' or **d** satisfies '(∃x)Gx'.

Suppose the former holds. Then for some member of the UD **u**, **d**[**u**/y] satisifies 'Fy'. In that case, **d**[**u**/y] satisifies 'Fy ∨ (∃x)Gx'. Therefore, **d** satisfies '(∃y)(Fy ∨ (∃x)Gx)'.

Now suppose that the latter holds. Because 'y' does not occur in '(∃x)Gx', and because **d** satisfies that sentence, **d**[**u**/y] satisfies that sentence for some choice of **u**. Therefore, **d**[**u**/y] satisfies the disjunction 'Fy ∨ (∃x)Gx'. It follows that **d** satisfies '(∃y)(Fy ∨ (∃x)Gx)'.

In both cases, then, **d** satisfies '(∃y)(Fy ∨ (∃x)Gx)', Since the choice of **d** was arbitrary, it is satisfied by all variable assignments and thus is true. So if the top sentence is true, so is the bottom sentence.

Now suppose the bottom sentence is true on **I**. Then it is satisfied by an arbitrary **d**. In that case, for some member **u** in the UD of **I**, **d**[**u**/y] satisfies 'Fy ∨ (∃x)Gx'. Thus, either **d**[**u**/y] satisfies 'Fy' or it satisfies '(∃x)Gx'.

In the first case, **d** satisifes '(∃y)Fy', and so it satisfies '(∃y)Fy ∨ (∃x)Gx'.

In the second case, **d** satisfies '(∃x)Fx', since the assignment to 'y' is irrelevant, and so it satisisfies '(∃y)Fy ∨ (∃x)Gx'.

Therefore, under both assumptions, **d** satisfies '(∃y)Fy ∨ (∃x)Gx'. So **d** satisfies the sentence. Since the choice of **d** is aribitrary, the sentence is true. So, if the bottom sentence is true, so is the top one.

Name________________________________

4. Show that the following sentence is quantificationally indeterminate by constructing an interpretation on which it is true and one on which it is false. State, using either the formal or the informal semantics, why it is true on the interpretation on which it is true, and why it is false on the interpretation on which it is false. (15 points)

(∃x)((Fx & (∀y)(Fy ⊃ x = y)) & Gx)

UD: {1}
I(F): {⟨**u**⟩: **u** is odd}
I(G): {⟨**u**⟩: **u** is evenly divisible by 1}

We know from arithmetic that ⟨1⟩ is a member of **I**(F) and **I**(G). So **d**[1/x] satisfies both 'Fx' and 'Gx'. The variable assignment **d**[1/x, 1/y] satisfies 'x = y', since $\text{den}_{\mathbf{I},\mathbf{d}[1/x,1/y]}(x) = \text{den}_{\mathbf{I},\mathbf{d}[1/x,1/y]}(y)$. And so it satisfies 'Fy ⊃ x = y'. Since 1 is the only object in the UD, **d**[1/x] satisfies '(∀y)(Fy ⊃ x = y)'. Therefore, **d**[1/x] satisfies 'Fx & (∀y)(Fy ⊃ x = y)'. It follows that **d**[1/x] satisfies '(Fx & (∀y)(Fy ⊃ x = y)) & Gx'. And so, because the number 1 is the only member of the UD, **d** satisfies ((∀x)(Fx & (∀y)(Fy ⊃ x = y)) & Gx). Since the choice of **d** is arbitrary, all variable assignments satisfy the sentence, and it is true.

UD: {1}
Fx: x is even
Gx: x is evenly divisible by 2

We know from arithmetic that the number 1 is not even (and not evenly divisible by 2). Because 1 is not even, 1 does not satisfy the condition specified by 'Fx'. Therefore, it does not satisfy the condition specified by 'Fx & (∀y)(Fy ⊃ x = y)'. Further, it does not satisfy the condition specified by '(Fx & (∀y)(Fy ⊃ x = y) & Gx)'. Therefore, it is not the case that at least one member of the UD does meets the condition specified by the open sentence following the existential quantifier, so the sentence '(∃x)((Fx & (∀y)(Fy ⊃ x = y)) & Gx)' is false on this interpretation.

5. Using the formal semantics for *PL*, determine the truth-value of the following sentence on the interpretation given. Show in detail how the truth-value is determined. (15 points)

$(\forall x)(\forall y)(Axxy \supset Gyx)$

UD: The set of all positive integers
$\mathbf{I}(G)$ is $\{<\mathbf{u}_1,\mathbf{u}_2>: \mathbf{u}_1 \text{ is greater than } \mathbf{u}_2\}$
$\mathbf{I}(A)$ is $\{<\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3>: \mathbf{u}_1 \text{ plus } \mathbf{u}_2 \text{ equals } \mathbf{u}_3\}$

The sentence is true on the interpretation given. Informally, the reason is that the sum of any positive integer with itself is always greater than that number. E.g., 3+3 = 6, and 6 > 3.

Consider an arbitrary variable assignment $\mathbf{d}$. Let $\mathbf{d}(x) = \mathbf{u}_1$ and $\mathbf{d}[\mathbf{u}_1/x](y) = \mathbf{u}_2$. Either $\langle \mathbf{u}_1,\mathbf{u}_1,\mathbf{u}_2\rangle$ is a member of $\mathbf{I}(A)$ or it is not.

If it is not, then $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$ does not satisfy 'Axxy' and so it does satisfy 'Axxy ⊃ Gyx'.

If it is a member of the extension of 'A', then $\mathbf{u}_2$ is the sum of a number $\mathbf{u}_1$ added to itself, and we know from arithmetic that it is larger than the positive integer $\mathbf{u}_1$ that is doubled to form it. So, $\langle \mathbf{u}_2,\mathbf{u}_1\rangle$ is a member of $\mathbf{I}(G)$. In that case, $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$ satisfies 'Gyx', and so it satisfies the conditional 'Axxy ⊃ Gyx'.

On either of the two suppositions, 'Axxy ⊃ Gyx' is satisfied by $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$. The choice of $\mathbf{u}_2$ was arbitrary, so all such values satisfy the condtional, in which case $\mathbf{d}[\mathbf{u}_1/x]$ satisfies '(∀y)(Axxy ⊃ Gyx)'. Similarly, the choice of $\mathbf{u}_1$ was arbitrary, so $\mathbf{d}$ satisfies '(∀x)(∀y)(Axxy ⊃ Gyx)'. All choices of $\mathbf{d}$ satisfy the sentence, so it is true on the present interpretation.

Name________________________________

6. Using either informal or formal semantics, show that the following sentence is quantificationally true in *PLI.* (15 points)

$(\forall x)(\forall y)(x = y \supset (Fx \equiv Fy))$

Let $\mathbf{d}$ be an arbitrary variable assignment based on an arbitrary interpretation $\mathbf{I}$. Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be arbitrary members of the UD. Either $\mathbf{u}_1$ is identical to $\mathbf{u}_2$ or it is not.

If $\mathbf{u}_1$ is not identical to $\mathbf{u}_2$, then $\text{des}_{\mathbf{I},\mathbf{d}[\mathbf{u}_1/x,\mathbf{u}_2/y]}(x) \neq \text{des}_{\mathbf{I},\mathbf{d}[\mathbf{u}_1/x,\mathbf{u}_2/y]}(y)$. In that case, the antecedent of '$x = y \supset (Fx \equiv Fy)$' is not satisfied by $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$, so the conditional itself is satisfied by that variable assignment.

If $\mathbf{u}_1$ is identical to $\mathbf{u}_2$, then $\mathbf{u}_1$ is a member of $\mathbf{I}(F)$ if and only if $\mathbf{u}_2$ is a member of $\mathbf{I}(F)$. So either both 'Fx' is satisfied by $\mathbf{u}_1$ and 'Fy' is satisfied by $\mathbf{u}_2$, or both 'Fx' is not satisfied by $\mathbf{u}_1$ and 'Fy' is not satisfied by $\mathbf{u}_2$. Therefore, '$Fx \equiv Fy$' is satisifed by $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$. In that case, the conditional '$x = y \supset (Fx \equiv Fy)$' is satisfied by $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$.

In either case, then, $\mathbf{d}[\mathbf{u}_1/x, \mathbf{u}_2/y]$ satisfies '$x = y \supset (Fx \equiv Fy)$'. Therefore, $\mathbf{d}[\mathbf{u}_1/x]$ satisfies '$(\forall x)(x = y \supset (Fx \equiv Fy))$', since the choice of $\mathbf{u}_2$ was arbitrary. By the same reasoning for $\mathbf{u}_1$, $\mathbf{d}$ satisfies '$(\forall x)(\forall y)(x = y \supset (Fx \equiv Fy))$'. Since the choice of $\mathbf{d}$ was arbitrary, the sentence is satisfied by all variable assignments, so it is true on $\mathbf{I}$. And since the choice of $\mathbf{I}$ was arbitrary, the sentence is true on all interpretations; that is, it is quantificationally true, which was to be proved.

Name________________________________

7. Prove that the following argument is valid in *PD*. (10 points)

$\sim(\exists x)(Fx \ \& \ Gx)$

$(\forall x)(Fx \supset \sim Gx)$

1	$\sim(\exists x)(Fx \ \& \ Gx)$	Assumption
2	Fa	Assumption
3	Ga	Assumption
4	$Fa \ \& \ Ga$	2 3 &I
5	$(\exists x)(Fx \ \& \ Gx)$	4 $\exists$I
6	$\sim(\exists x)(Fx \ \& \ Gx)$	1 R
7	$\sim Ga$	3-6 $\sim$I
8	$Fa \supset \sim Ga$	2-7 $\supset$I
9	$(\forall x)(Fx \supset \sim Gx)$	8 $\forall$I