Second Midterm Solutions
Philosophy 112
Winter 2003

Please work all the problems in the space provided. Each problem is worth 20 points. You may use only the rule set noted on the individual problems, except that you may use the *falsum* rule on any problem. Please note that with *PD+* and *PDI*, you are not required to use any of extra rules provided by those rule-sets, though in the case of *PDI*, the use of identity rules *may* be unavoidable.

1. Prove that the following is a theorem of *PD*.

$(\forall x)Fx \vee (\exists x) \sim Fx$

1	$\sim((\forall x)Fx \vee (\exists x)\sim Fx)$	Assumption
2	$\sim Fa$	Assumption
3	$(\exists x)\sim Fx$	2 $\exists$ I
4	$(\forall x)Fx \vee (\exists x)\sim Fx)$	3 $\vee$ I
5	$\sim((\forall x)Fx \vee (\exists x)\sim Fx)$	1 R
6	Fa	2-5 $\sim$ E
7	$(\forall x)Fx$	6 $\forall$ I
8	$(\forall x)Fx \vee (\exists x)\sim Fx$	7 $\vee$ I
9	$(\forall x)Fx) \vee (\exists x)\sim Fx$	1-8 $\sim$ E

2. Prove the equivalence of the following two sentences of *PD*.

∼(∀x)Fx
(∃y)∼Fy

1	∼(∀x)Fx	Assumption
2	∼(∃y)∼Fy	Assumption
3	∼Fa	Assumption
4	(∃y)∼Fy	3 ∃ I
5	∼(∃y)∼Fy	2 R
6	Fa	2-5 ∼ E
7	(∀x)Fx	6 ∀ I
8	∼(∀x)Fx	1 R
9	(∃y)∼Fy	1-8 ∼ E

1	(∃y)∼Fy	Assumption
2	∼Fa	Assumption
3	(∀x)Fx	Assumption
4	Fa	3 ∀ E
5	∼Fa	2 R
6	∼(∀x)Fx	2-5 ∼ I
7	∼(∀x)Fx	1 2-6 ∃ E

3. Prove that the following derivability relation holds in *PD+*.

$\{(\forall x)(Qx \vee Ixb)\} \vdash (\exists x)Qx \vee (\forall x)Ixb$

1	$(\forall x)(Qx \vee Ixb)$	Assumption
2	$\quad \sim((\exists x)Qx \vee (\forall x)Ixb)$	Assumption
3	$\quad \sim(\exists x)Qx \ \& \sim(\forall x)Ixb$	2 DeM
4	$\quad \sim(\exists x)Qx$	3 &E
5	$\quad (\forall x)\sim Qx$	4 QN
6	$\quad \sim Qa$	5 ∀ E
7	$\quad Qa \vee Iab$	1 ∀ E
8	$\quad Iab$	6 7 DS
9	$\quad (\forall x)Ixb$	8 ∀ I
10	$\quad (\exists x)Qx \vee (\forall x)Ixb$	9 ∨ I
11	$(\exists x)Qx \vee (\forall x)Ixb$	2-10 ∼ E

4. Show that the following argument is valid in *PDI*.

The current President of the United States was born in Connecticut. Connecticut and Texas are different, and nothing has more than one birthplace. Therefore, no current president of the United States was born in Texas.

$(\exists x)((Pxu \mathbin{\&} (\forall y)(Pyu \supset x = y)) \mathbin{\&} Bxc)$
$\sim c = t \mathbin{\&} (\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z))$

$\sim(\exists x)(Pxu \mathbin{\&} Bxt)$

1	$(\exists x)((Pxu \mathbin{\&} (\forall y)(Pyu \supset x = y)) \mathbin{\&} Bxc)$	Assumption
2	$\sim c = t \mathbin{\&} (\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z))$	Assumption
3	$(\exists x)(Pxu \mathbin{\&} Bxt)$	Assumption
4	$(Pau \mathbin{\&} (\forall y)(Pyu \supset a = y)) \mathbin{\&} Bac$	Assumption
5	Bac	4 & E
6	$Pau \mathbin{\&} (\forall y)(Pyu \supset a = y)$	4 & E
7	$(\forall y)(Pyu \supset a = y)$	6 & E
8	$(\forall x)(\forall y)(\forall z)(Bxy \supset (Bxz \supset y = z))$	2 & E
9	$(\forall y)(\forall z)(Bay \supset (Baz \supset y = z))$	8 $\forall$ E
10	$(\forall z)(Bac \supset (Baz \supset c = z))$	9 $\forall$ E
11	$Bac \supset (Bat \supset c = t)$	10 $\forall$ E
12	$Bat \supset c = t$	5 111 $\supset$ E
13	$Pru \mathbin{\&} Brt$	Assumption
14	Pru	13 & E
15	Brt	13 & E
16	$Pru \supset a = r$	7 $\forall$ E
17	$a = r$	14 16 $\supset$ E
18	Bat	15 17 = E
19	$c = t$	12 18 $\supset$ E
20	$\sim c = t$	2 & E
21	$c = t \mathbin{\&} \sim c = t$	19 20 & I
22	$c = t \mathbin{\&} \sim c = t$	3 13-21 $\exists$ E
23	$c = t \mathbin{\&} \sim c = t$	1 4-22 $\exists$ E
24	$c = t$	23 & E
25	$\sim c = t$	23 & E
26	$\sim(\exists x)(Pxu \mathbin{\&} Bxt)$	3-25 $\sim$ E

5. Using a symbolization key that reveals as much logical structure as possible, symbolize the following sentences in *PL*. Then prove that the set of *PL* sentences is inconsistent in *PD+*.

Only an earlier event causes an event. One event is earlier than another only if the later event is not earlier than the earlier one. Some event causes every event.

UD: The set of all events
Exy: x is earlier than y
Cxy: x causes y

$(\forall x)(\forall y)(Cxy \supset Exy)$
$(\forall x)(\forall y)(Exy \supset \sim Eyx)$
$(\exists x)(\forall y)Cxy$

1	$(\forall x)(\forall y)(Cxy \supset Exy)$	Assumption
2	$(\forall x)(\forall y)(Exy \supset \sim Eyx)$	Assumption
3	$(\exists x)(\forall y)Cxy$	Assumption
4	$\quad (\forall y)Cay$	Assumption
5	$\quad Caa$	4 ∀ E
6	$\quad (\forall y)(Cay \supset Eay)$	1 ∀ E
7	$\quad Caa \supset Eaa$	6 ∀ E
8	$\quad Eaa$	5 7 ⊃ E
9	$\quad (\forall y)(Eay \supset \sim Eya)$	2 ∀ E
10	$\quad Eaa \supset \sim Eaa$	8 ∀ E
11	$\quad \sim Eaa$	8 10 ⊃ E
12	$\quad \bot$	8 11 ⊥ I
13	$\bot$	3 4-12 ∃ E
14	$\sim(\exists x)(\forall y)Cxy$	13 ⊥ E