Formal Semantics for Atomic Sentences

Interpretations

- In giving the formal semantics for atomic sentences of Predicate Logic, we must introduce a number of new metavariables.
  - ‘**I**’ designates an interpretation.
  - ‘**D**’ designates a set containing at least one object (the domain of the interpretation).
  - ‘**v**’ designates a (“valuation”) function which gives the designation of constant terms, the extensions of predicates, and the truth-values of sentence letters.
  - ‘***f***’, ‘***g***’, ‘***h***’ designate function symbols.
- **I** is an ordered pair ⟨**D**,**v**⟩.

Specifying the Domain

- We may specify the domain by enumerating its members:
  - **D** = {Adam, Eve}
  - **D** = {1, 2, 3, 4, 5}

  A domain may also be specified by description:
  - **D** = the set of all positive integers
  - **D** = {x: x is a positive integer}

  More generally, we may refer to objects in the domain using the metavariables $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, ...

The Valuation Function

- In the following interpretation I = {**D**,**v**}, the valuation function‘**v**’ assigns truth-values to sentence letters.
  - **v**(P) = t
  - **v**(Q) = f

  v assigns members of the domain to names.
  - **v**(a) = Adam
  - v(e) = Eve
- **v** assigns extensions to predicates.

- $\mathbf{v}$(B) = {⟨Adam⟩}
- $\mathbf{v}$(B) = {x: x is blond}
- $\mathbf{v}$(L) = {⟨Adam, Eve⟩, ⟨Eve, Adam⟩}
- $\mathbf{v}$(L) = {⟨x, y⟩: x loves y}

Valuations of Function Symbols

- The valuation function assigns a value to each function symbol (without parentheses).
- For a function symbol of n places, the value is a set of n+1-tuples.
  - Consider the two-place addition function and a domain consisting of the positive integers.
  - $\mathbf{v}(f) = \{\langle 1, 1, 2\rangle, \langle 1, 2, 3\rangle, \langle 1, 3, 4\rangle, \ldots\}$.
- The first n places specify the values of the argument terms.
- The last place specifies the value of the function.

$$\begin{array}{ccc} \langle\ 1, & 1, & 2\rangle \\ \downarrow & \downarrow & \downarrow \\ \text{argument} & \text{argument} & \text{value} \end{array}$$

Values of Filled-in Function Symbols

- The value assigned to the function symbol allows the determination of the value of a filled-in symbol.
- Let $\boldsymbol{f}$ be a two place function.
- Then $\mathbf{v}(\boldsymbol{f})$ is a set of ordered triples.
- $\mathbf{v}(\boldsymbol{f}(\mathbf{t}, \mathbf{u}))$ is the last member of the the ordered triple whose first two members are $\mathbf{v}(\mathbf{t})$ and $\mathbf{v}(\mathbf{u})$.
  - Let $\mathbf{v}(f)$ be the addition function as specified above.
  - Let $\mathbf{v}$(a) = 1.
  - Then $\mathbf{v}(\boldsymbol{f}(\text{a,a}))$ is the last member of $\langle \mathbf{v}(\text{a}), \mathbf{v}(\text{a}), 2\rangle$, which is the same as $\langle 1, 1, 2\rangle$, and therefore is 2.

Symbols for Set Membership and Non-Membership

- It is convenient to have a symbol indicating membership in a set.
- The symbol '$\in$' indicates set-membership.
  - Adam $\in$ {Adam, Eve}
- The membership or "belongs to" symbol when struck through indicates non-membership.
  - Cid $\notin$ {Adam, Eve}

Truth-Definition for Atomic Sentences

- Let **P** be an n-place predicate.
- Let $\mathbf{t}_1$, $\mathbf{t}_2$, . . ., $\mathbf{t}_n$ be constant terms.
- Let $\mathbf{Pt}_1\mathbf{t}_2 \ldots \mathbf{t}_n$ be an atomic sentence of Predicate Logic consisting of the n-place predicate **P** followed by n constant terms $\mathbf{t}_1$, $\mathbf{t}_2$, . . ., $\mathbf{t}_n$, in that order.

Truth-Definition for Atomic Sentences of Predicate Logic
$\mathbf{v}(\mathbf{Pt}_1\mathbf{t}_2 \ldots \mathbf{t}_n)$ = t if $\langle \mathbf{v}(\mathbf{t}_1), \mathbf{v}(\mathbf{t}_2) \ldots \mathbf{v}(\mathbf{t}_n)\rangle \in \mathbf{v}(\mathbf{P})$
$\mathbf{v}(\mathbf{Pt}_1\mathbf{t}_2 \ldots \mathbf{t}_n)$ = f otherwise

- Compare this definition with the informal definition given earlier.
- An atomic sentence with an n-place predicate is true in an interpretation if the n-tuple consisting of the referents of the arguments (from left to right) is a member of the extension of the predicate; the sentence is false otherwise.

An Example

- Let **I** = {**D**, **v**}.
- **D** = {Adam, Eve}
- **v**(a) = Adam
- **v**(e) = Eve
- **v**(L) = {$\langle$Adam, Eve$\rangle$, $\langle$Eve, Adam$\rangle$}
- So, $\langle$Adam, Eve$\rangle \in$ **v**(L)
- So, $\langle$**v**(a), **v**(e)$\rangle \in$ **v**(L)
- So, 'Lae' is true in **I**.

Another Example

- Let **I** = {**D**, **v**}.
- **D** = the set of all positive integers.
- **v**(o) = 1
- **v**(t) = 2
- **v**(f) = {⟨1, 1, 2⟩, ⟨1, 2, 3⟩, . . .} [the addition function]
- **v**(G) = {⟨x,y⟩ x > y}
- So, **v**(f(o,o)) = 2 [1 + 1 = 2]
- So, ⟨2, 2⟩ $\notin$ v(G)
- So, ⟨**v**(t), **v**(f(o,o))⟩ $\notin$ **v**(G)
- So, ‘Gtf(o,o)’ is false in I.