

Exercises for Functions and Identity

January 14, 2005

1 Exercises FI-1

State whether or not the following expressions are sentences of Predicate Logic. If not, state why not.

- a) $f(a)$
- b) $f(a) = \sim b$
- c) $\sim f(a) = a$
- d) $Lf(b)f(a)$.
- e) $Lf(f)a$
- f) $Babcdef(a)$
- g) $Bf(a) = b$
- h) $f(f(f(a))) = f(a)$

2 Exercises FI-2

Given the following transcription guide, determine whether the following sentences are true or false.

D = the set of all positive integers

o: 1

w: 2

t: 3

f: the addition function

s: the successor function [e.g., the successor of 1 is 2]

Gxy: x is greater than y

- a) $Gws(o)$
- b) $f(o,w) = t$
- c) $s(s(o)) = s(w)$
- d) $s(w) = f(o,w)$
- e) $Gf(s(o),s(o))w$

3 Exercises FI-3

Using the above transcription guide, transcribe the following sentences into Predicate Logic.

- a) $1 + 3 > 1 + 2$
- b) The successor of 1 is not greater than the successor of 2.
- c) The successor of 1 is greater than or equal to 1.

4 Exercises FI-4

Show that the following derivability claims hold, using Sentence Logic rules plus the introduction and elimination rules for '='.

- a) $\{a = b, b = c\} \vdash a = c$
- b) $\{a = b\} \vdash Ka \equiv Kb$
- c) $\{Lab, b = f(a), c = f(b), Lf(b)c\} \vdash Laf(a) \& Lcc$
- d) $\{\sim a = c\} \vdash \sim a = b \vee \sim b = c$ [more challenging]

5 Exercises FI-5

Show that the following property ["Substitutivity of Identicals"] holds in the semantics for Predicate Logic. Assume the antecedent of the conditional and give an argument for why the consequent follows.

If 'Lea' and 'a = e' are true in an interpretation, then 'Lee' and 'Laa' are true in that interpretation.