Object Language and Metalanguage

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Formal Languages

- Sentence Logic and Predicate Logic are *formal languages*.

- A formal language is a set of *sentences* generated by *rules of formation* from a *vocabluary*.

- The sentences of Sentence Logic and Predicate Logic are not part of natural language (though some may resemble natural-language sentences).

- The formal languages Sentence Logic and Predicate Logic are the objects of our study, and as such they are called *object languages*.

The Metalanguage

- If we are going to state anything about an object language, we must make use of a language.

- We call a language used to study an object language a *metalanguage*.

- In theory, the metalanguage may be identical or include the object language.

  - In some parts of linguistics, English is used to describe features of the English language.

- We will strictly separate our metalanguage (English with some extra technical vocabulary) from our object languages.

- Keeping the languages separate allows us to avoid paradoxes of self-reference, such as those described by Tarski.

Use and Mention

- When we employ the metalanguage to refer an item of language, we are said to *mention* that item of language.

- To mention an item in the object language, one places it within single quotation marks.

- – 'Obama' has five letters and starts with an 'O.'
- – 'Barack Obama was born in Ohio' is a false sentence.
- – 'This sentence is false' is true.

- In each of the examples, English as the metalanguage is *used* to mention words or sentences of English.

- In the sentence 'Barack Obama is president,' 'Barack Obama' is *used* to refer to the president.

Metavariables

- We may also use English as a metalanguage to mention items of Sentence Logic and Predicate Logic.

  - – '⊃' is a connective of Sentence Logic.
  - – 'P ⊃ Q' is a conditional.
  - – If 'P' and 'P ⊃ Q' have the value t, then 'Q' has the value t.

- To state general facts about Sentence Logic and Predicate Logic, we must use expressions that designate *classes* of items of the object language.

- Such expressions are called *metavariables*.

Metavariables for Sentences and Sets of Sentences

- To mention metavariables themselves, single quotation marks must be used.

- '**Q**' through '**Z**' will be used as metavariables for sentences of both Sentence Logic and Predicate Logic.

- '**X**' through '**Z**' will be used as metavariables for sets of sentences of both Sentence Logic and Predicate Logic.

Metavariables and Connectives

- In English, we refer to connectives of Sentence Logic using such expressions as 'sign of negation' and 'sign of the conditional.'

- We may also refer to them by putting single quotation marks around the symbols for the connectives.

  - – '∼.'
  - – '⊃.'

- For conciseness, we must have a way to combine our use of metavariables with reference to connectives, to avoid awakward expressions like the following:

  - – If **X** is a sentence, then the sentence **X** preceded by '∼,' with the resulting expression enclosed in parentheses, is a sentence.

Names of Themselves

- We would like to be able to combine the use of metavariables with the use of connectives.

    – If **X** is a sentence, then ~**X** is a sentence.

- But such a sentence of the metalanguage includes an item of the object language which is not enclosed in parentheses.

- We will permit such usage (following Carnap) by declaring that the connectives are names, in the metalanguage, of themselves in the object language.

- Names which name themselves are said to be used *autonymously*.

Metavariables for Predicate Logic

- The vocabulary of Predicate Logic is an *extension* of the vocabluary of Sentence Logic.

- It contains all the expressions of the vocabulary of Sentence logic, along with other expressions which are proper to Predicate Logic.

- The metavariables for these new vocabulary items will be introduced when the vocabulary items themselves are introduced.

Sets

- An important part of the metalanguage for Sentence Logic and Predicate Logic is set-theoretic notation.

    – We have already described metavariables such as '*X*' that refer to sets of sentences in the object language.

- Sets are indicated by enclosing the names of their members in curly brackets '{' and '}.'

- '{'P,' 'P ⊃ Q'}' indictes the set consisting of the sentences 'P' and 'P ⊃ Q.'

- *Ordered sets* are indicated by enclosure in angle brackets '⟨' and '⟩.'

    – Thus, '⟨'P', 'P ⊃ Q' ⟩' indicates the set consisting of 'P' and 'P ⊃ Q,' in that order.

Specifying Sets

- There are two methods of specifying the contents of a set.

- The method of *enumeration* simply lists the contents of the set using linguistic items that refer to the objects in that set.

  - {The President of the United States, Timothy Geithner, 2010}.
  - ⟨1,2⟩

- The method of *description* states what conditions must be satisfied for something to be a member of the set.

  - {x: x is even} (the set of all x such that x is even, or the set of all that is even).
  - {⟨x, y⟩: x > y} (the set of all ordered pairs x and y such that x is greater than y.