Validity in Predicate Logic

Valid Arguments in Natural Language

- Arguments in natural language consist of a set of sentences serving as **premises** and a single sentence serving as the **conclusion**.
- A natural language argument is valid if and only if it is not possible for all the premises to be true and the conclusion false.
- Validity of natural language arguments can be evaluated by transcribing them into Predicate Logic and applying the semantics to the transcribed arguments.

Valid Arguments in Predicate Logic

- Truth and satisfaction in an interpretation are the most basic semantical properties of sentences of Predicate Logic.
- These properties can be used to determine the truth-value, in an interpretation, of a Predicate Logic sentence (conclusion) relative to a set of Predicate Logic sentences (premises) in an argument of Predicate Logic.
- The goal is to determine whether there is an interpretation in which all the premise-sentences have the value t and the conclusion-sentence has the value f.
- If there is such an interpretation, it is a **counterexample**, and the transcribed argument is **invalid**.
- If there are no counterexamples, then transcribed argument is **valid**.

Determining Invalidity

- To show that an argument of Predicate Logic is invalid, one produces an interpretation to serve as a counterexample.
- Producing a counterexample requires the specification of a domain, as well as the designations of the names and function symbols, and the extensions of the predicates occurring in the sentence.
- Premises: $(\exists x)Fx$, $(\exists x)Gx$ Conclusion: $(\exists x)(Fx \,\&\, Gx)$
- To show the invalidity of this argument, we produce an interpretation which makes the conclusion false, making sure that it allows the premises to be true.

An Example

- D = {1, 2}, v(F) = {⟨1⟩}, v(G) = {⟨2⟩}
- No variable assignment to ‘x’ satisfies both ‘Fx’ and ‘Gx’, and so none satisfies ‘Fx & Gx’, so ‘(∃x)(Fx & Gx)’ is false.
- d[1/x] satisfies ‘Fx’, so any d satisfies ‘(∃x)Fx
- d[2/x] satisfies ‘Gx’, so any d satisfies ‘(∃x)Gx’; so, both premises are true.
- So on this interpretation, the premises are true and the conclusion false, which demonstrates the invalidity of the argument.

Determining Validity

- Because validity of arguments is defined in terms of all possible interpretations, it cannot be proved on the basis of a single interpretation.
- General reasoning about interpretations is required.
- For this reason, we use metavariables to indicate arbitrary:
  - Interpretations **I**
  - Domains **D**
  - Objects in the domain **u** (with or without positive integer subscripts)
  - Valuation functions **v**
- At this level of generality, we can still draw conclusions about what must hold if the premises of an argument are to be true in an arbitrary interpretation.

An Example

- To prove: {(∀x)(Fx ⊃ Gx), Fa} ⊨ Ga
- Let **I** be an arbitrary interpretation, **v** a valuation function in **I**, and **d** an arbitrary variable assignment.
- Suppose that ‘(∀x)(Fx ⊃ Gx)’ and ‘Fa’ are true in **I**.
- Let **v**(a) = $\mathbf{u}_1$
- Then for all **u** in the domain **D** of **I**, **d**[**u**/x] satisfies ‘Fx ⊃ Gx’.
- Because **d** satisfies ‘Fa’, ⟨**v**(a)⟩ ∈ **v**(F), so ⟨$\mathbf{u}_1$ ⟩ ∈ **v**(F).
- Then **d**[$\mathbf{u}_1$/x] satisfies ‘Fx’, so **d**[$\mathbf{u}_1$/x] satisfies ‘Gx.’
- It follows that ⟨$\mathbf{u}_1$ ⟩ ∈ **v**(G), so ⟨**v**(a)⟩ ∈ **v**(G).
- Then **d** satisfies ‘Ga’, which is thus true in **I**, QED.