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**Final Examination**  
**Philosophy 134**  
**Spring, 2001**

1. Show, using the derivation rules for  $B$ , that the following derivability relation holds in  $B$ :

$$\{\Diamond\Box P\} \vdash_B P$$

2. Show, using the derivation rules for  $S5$ , that the following derivability relation holds in  $S5$ :

$$\{\Diamond\Box P\} \vdash_{S5} \Box\Diamond P$$

3. Show, using the semantics for  $S_4$ , that the following sentence is not  $S_4$ -valid:

$$\not\models_{S_4} \Box(\Box P \supset Q) \vee \Box(\Box Q \supset P)$$

4. Show, using the semantics for  $S5$ , that the following semantical entailment fails in  $S5$ :

$$\{\Box\Diamond P\} \not\models_{S5} \Diamond\Box P$$

5. Explain why truth-value gaps are introduced for systems  $QPL$ . How can a semantics with truth-value gaps avoid invalidating  $PL$  consequences? How does free logic allow avoidance of truth-value gaps?

6. Using the derivational rules for *Q1-S5*, show that the following sentence is a theorem of *Q1-S5*:

$$\vdash_{S5} \sim \Box((\forall x)\Diamond\Box Fx \ \& \ (\exists x) \sim Fx))$$

7. Show, using the semantics for  $Q1-S5$ , that the following entailment fails in the semantical system  $Q1-S5$ :

$$\{(\exists x)\sim \Box Fx\} \not\vdash_{Q1-S5} \Diamond(\forall x)\sim Fx$$

8. Suppose we have the following *Q1-D* interpretation.

$\mathbf{R}w_1w_2, \mathbf{R}w_2w_3, \mathbf{R}w_3w_1$

$\mathbf{D}=\{1, 2, 3\}$

$\mathbf{v}(F, w_1)=\{\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,1 \rangle\}$

$\mathbf{v}(F, w_2)=\{\langle 2,3 \rangle, \langle 1,3 \rangle\}$

$\mathbf{v}(F, w_3)=\{\langle 1,3 \rangle\}$

Evaluate the following sentence at each world. Show why you give it the truth-values that you do.

$\diamond(\forall x)(\exists y)Fxy$